BALANCED GAIN AND LOSS IN SPATIALLY EXTENDED NON-$\mathcal{PT}$-SYMMETRIC MULTI-WELL POTENTIALS

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INTRODUCTION

Gain and loss in open quantum system can be modelled by using complex potentials. They are balanced if the corresponding non-Hermitian Hamiltonian has real energy eigenvalues. The most prominent class of non-Hermitian Hamiltonians that allows balanced gain and loss are $\mathcal{PT}$-symmetric Hamiltonians [1]. A promising candidate for realizing balanced gain and loss in a quantum system are Bose-Einstein condensates in multi-well optical potentials. A challenge here is, however, that to realize a $\mathcal{PT}$-symmetric potential, one has to adjust many parameters simultaneous very precisely. Fortunately there exists a class of asymmetric potentials where most parameters can be chosen arbitrarily and only a few have to be adjusted in order to achieve balanced gain and loss. In discrete matrix models they can be easily constructed using the concept of symmetrization [2]. Although the latter is not directly applicable to continuous systems, here we present a method that uses the similarity between continuous model and matrix model to construct Gaussian N-well potentials in such way that at least the first N eigenvalues are real or appear in complex conjugate pairs. We perform this explicitly for a double-well and a triple-well potential [3].

THEORY

A non-Hermitian Hamiltonian $\hat{H}$ is symmetrizable if there exist an operator $\hat{\eta}$ so that $\hat{\eta}\hat{H} = \hat{H}^\dagger\hat{\eta}$. Its spectrum then consist entirely of real and pairs of complex conjugate eigenvalues. If we consider a discrete system, i.e. a system with a finite dimensional Hilbert space, $\hat{H}$ and $\hat{\eta}$ are matrices. One can then easily adjust the parameters of $\hat{H}$ in such a way that it is symmetrizable [2]. However we are interested in the spectra of a continuous model, i.e. a system with infinite dimensional Hilbert space. Here it is also possible to construct $\hat{H}$ in such a way that it is symmetrizable, this however strictly limits the possible shape of the potential, for example one cannot have either only gain or only loss in each well. We want to know if a more physical potential like a Gaussian N-well potential of the form

$$V(x) = \sum_{n=1}^{N} (V_n + \mathrm{i}\Gamma_n) \exp\left(-\frac{(x_n - a_n)^2}{2\sigma_n^2}\right). \tag{1}$$

can have real eigenvalues. They are clearly not symmetrizable in general. However, if the wells are strongly localized, the system can be well approximated by a symmetrizable matrix model [3] with the eigenvalue equation $\boldsymbol{H}_{\text{eff}}\boldsymbol{d}_{\text{eff}} = \mu\boldsymbol{d}_{\text{eff}}$ with $H_{nn} = \varepsilon_n + \mathrm{i}\gamma_n$ and $H_{nn\pm} = -J$. It is thus reasonable to assume that we can construct the N-well potential in such a way, that it has at least N real or pair of complex conjugated eigenvalues.

METHOD

The naive approach to determine the potential parameters of (1) in a such way that the energy eigenvalues emerge real or in complex conjugate pairs is to find N functions of the potential parameters that become zero for such a spectrum and perform a root search of this functions. However, it turns out that this root search does not converge if the initial guess is not close to the actual solution. To obtain a good initial guess we use the following approach:

1. We choose the continuous model parameters in a way, that the potential roughly resemble a matrix model with balanced gain and loss.
2. We then manually fine tune the parameters of the continuous model until the corresponding matrix models spectrum has indeed the desired properties.
3. We then perform a root search with the continuous model parameters obtained in step 1. and 2. as an initial guess. The latter are close to the actual solution, so the root search will converge.

RESULTS

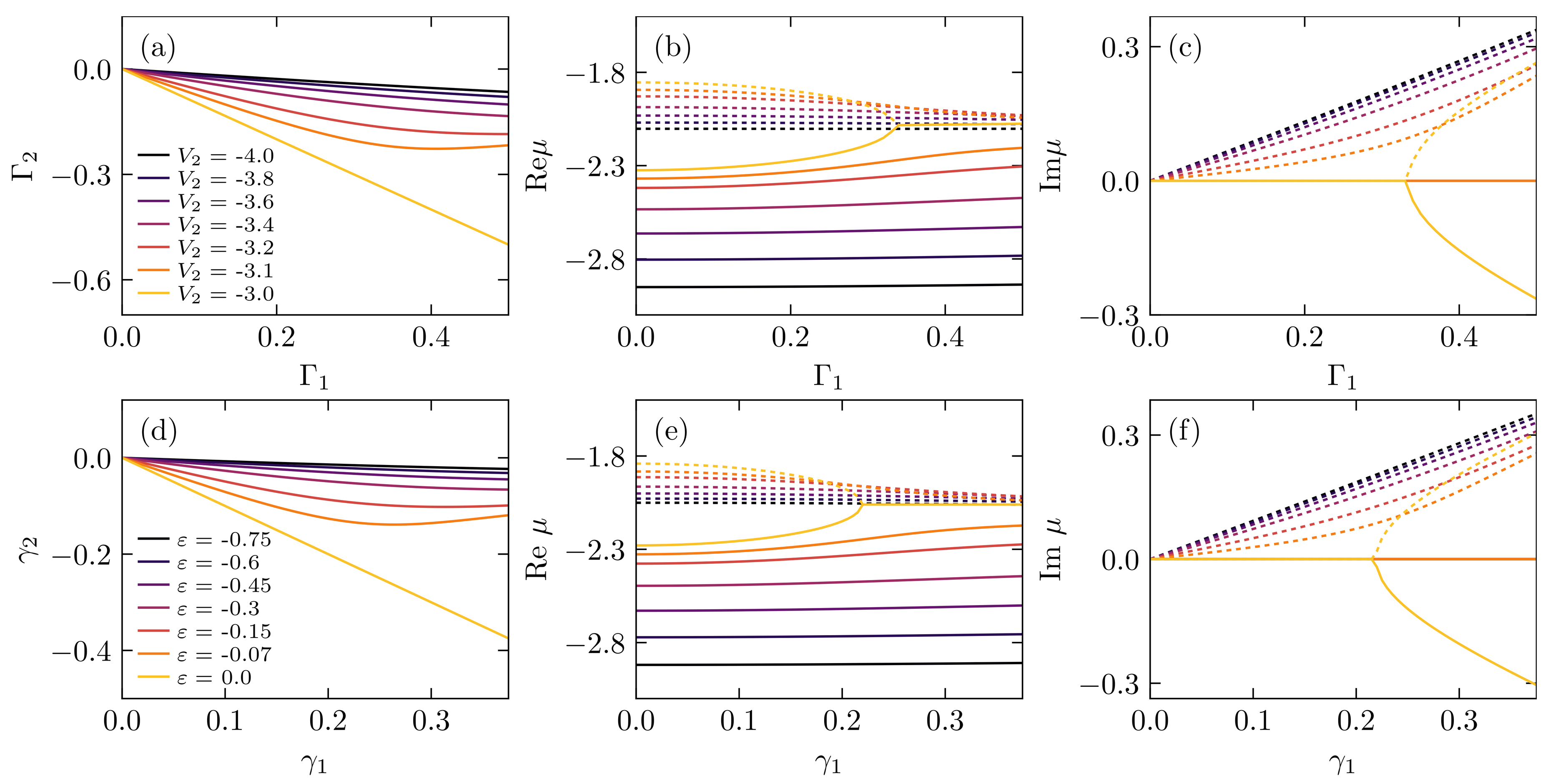

Figure 1: Double well potential. Only one real energy eigenvalue is possible.

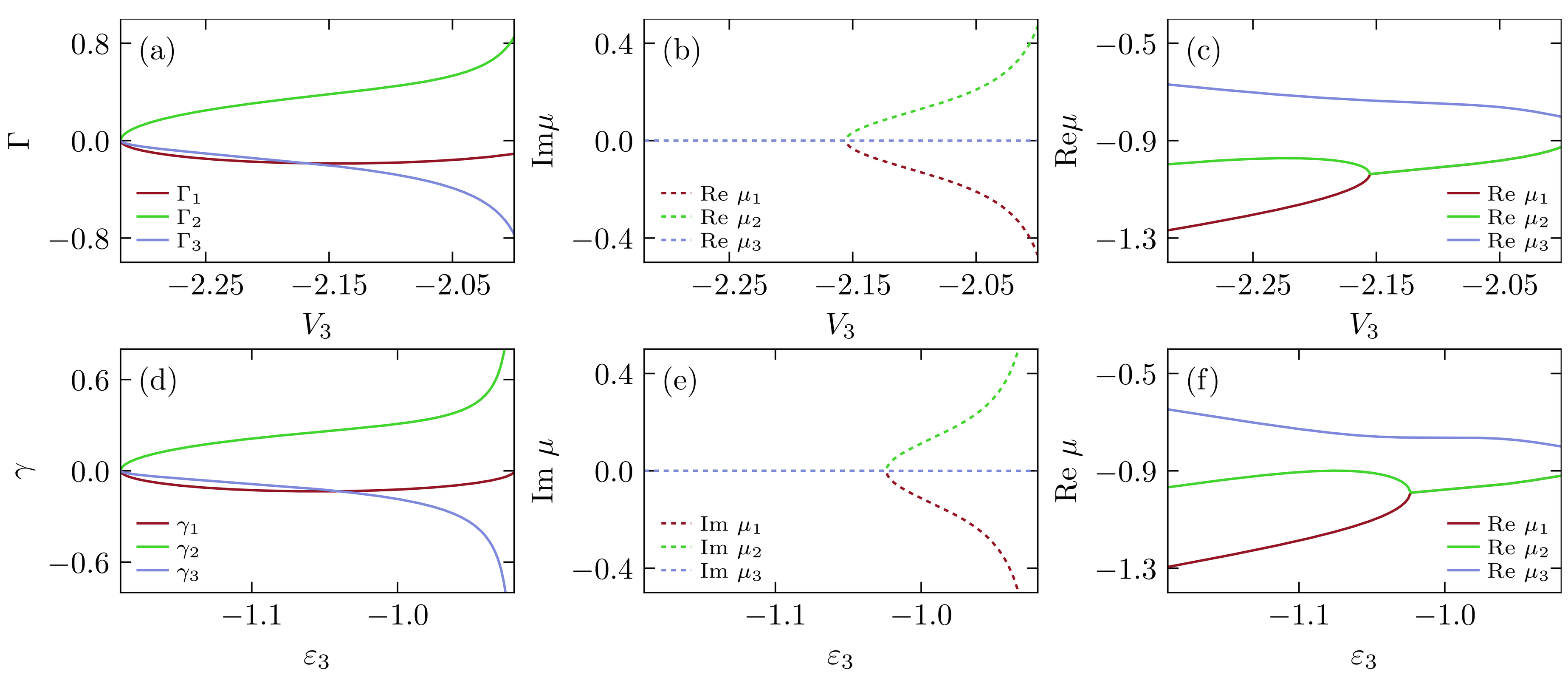

Figure 2: Triple well potential. We have either one or three real energy eigenvalue.

CONCLUSION

We use the symmetrizability of $(N \times N)$-matrix models [2] to developed a reliable method to construct assymetric complex N-well potentials which yield N real or pairs of complex conjugate eigenvalues and applied it explicitly to a double- and triple- well Gaussian potential.

REFERENCES

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