Resource Preservability, Thermodynamics, & Communication

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Introduction

Resource theory is a general, model-independent approach aiming to understand the qualitative notion of resource quantitatively [1]. In a given resource theory, free operations are physical processes that do not create resource and are considered zero-cost. This brings the natural question:

For a given free operation, what is its ability to preserve a resource?

We formulate this ability as resource preservability.

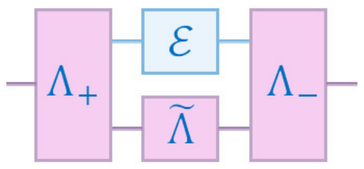
Goals | Quantifying resource preservability
Studying possible aplications

Formulation

A state resource theory can be written as a triplet $(R, \mathcal{F}_R, \mathcal{O}_R)$, which are

Resource	Free quantity	Free operation
R	$\mathcal{F}_R$	$\mathcal{O}_R$
The given resource	States that do not have the resource (free states)	Physical processes that do not generate the resource

A resource preservability theory is induced by a given state resource theory as follows

Resource	Free quantity	Free operation
The ability to preserve R	Channels in $\mathcal{O}_R$ such that they only output states in $\mathcal{F}_R$	Supperchannels of the following form
R-Preservability	R-annihilating channels Denoted by $\mathcal{O}_R^N$	 $\Lambda_\pm \in \mathcal{O}_R$; $\widetilde{\Lambda} \in \mathcal{O}_R^N$ s.t. $\widetilde{\Lambda} \otimes \Lambda \in \mathcal{O}_R^N$ $\forall \Lambda \in \mathcal{O}_R^N$ Free-superchanels

Now we can quantify resource preservability...

A function P_R is a R-preservability monotone if

$P_R(\mathcal{E}) \geq 0$ & equality holds iff $\mathcal{E} \in \mathcal{O}_R^N$

$P_R[F(\mathcal{E})] \leq P_R(\mathcal{E})$ $\forall$ free superchannel F

Quantification

We consider the following measure

$$P_D(\mathcal{E}_S) := \inf_{\Lambda_S \in \mathcal{O}_R^N} \sup_A D\left[(\mathcal{E}_S \otimes \widetilde{\Lambda}_A)(\rho_{SA}), (\Lambda_S \otimes \widetilde{\Lambda}_A)(\rho_{SA})\right]$$

where the optimization is taken over all possible ancillary systems (A), channels $\widetilde{\Lambda}_A$ in this system with property introduced earlier, joint input states ρ_{SA} , and R-annihilating channels Λ_S. Also, D is a distance measure satisfying

$D(\rho, \sigma) \geq 0$ & equality holds iff $\rho = \sigma$

$D[\mathcal{E}(\rho), \mathcal{E}(\sigma)] \leq D(\rho, \sigma)$ $\forall$channel $\mathcal{E}$

Result | P_D is a R-preservability monotone

Application to Thermo

Using max-relative entropy defined by $D_{\max}(\rho\|\sigma) := \log_2 \inf\{\lambda | \rho \leq \lambda\sigma\}$ [2] and set athermality as the resource (with the thermal state γ), we define

$$1 + \mathcal{B}^\epsilon(\mathcal{N}) := \sup_\rho \inf\Big\{n \mid \exists \mathcal{E}_\mathcal{C}\ s.t.\ \left\|\mathcal{E}_\mathcal{C}\big(\mathcal{N}(\rho) \otimes \gamma^{\otimes(n-1)}\big) - \gamma^{\otimes n}\right\|_1 < \epsilon\Big\}$$

where $\mathcal{E}_\mathcal{C}$ is a channel that can be realized by the thermalization model given in Ref. [3].

Result | Given a Gibbs-preserving channel $\mathcal{N}$ and $0 \leq \epsilon < 1$:

$$\mathcal{B}^\epsilon(\mathcal{N}) \leq \frac{1}{\epsilon^2} 2^{P_{D_{\max}}(\mathcal{N})}$$

If γ is full-rank, $\mathcal{N}$ is coherence-annihilating, and energy subspace condition [3] is satisfied, then

$$2^{P_{D_{\max}}(\mathcal{N})} \leq \mathcal{B}^\epsilon(\mathcal{N}) + \frac{2\sqrt{\epsilon}}{p_{\min}(\gamma)} + 1$$

where $p_{\min}(\gamma)$ is the smallest eigenvalue of γ.

As an application, this connects communication and thermodynamics:

Result | Assuming the same conditions as above, for $0 \leq \epsilon, \delta < 1$ we have

$$C_{(1)}^\delta(\mathcal{N}) \leq \log_2\left(\mathcal{B}^\epsilon(\mathcal{N}) + \frac{2\sqrt{\epsilon}}{p_{\min}(\gamma)} + 1\right) + \log_2 \frac{1}{1-\delta}$$

[1] E. Chitambar & G. Gour, Rev. Mod. Phys. **91**, 025001 (2019).
[2] N. Datta, IEEE Trans. Inf. Theory **55**, 2816 (2009).
[3] C. Sparaciari *et al*, Comm. Phys. **4**, 3 (2021).

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