The “Quantum Coherence and Entanglement for Quantum Technology” project is carried out within the First TEAM programme of the Foundation for Polish Science co-financed by the European Union under the European Regional Development Fund.

Coherence Resource in Quantum Computation

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We deal with the question of resource requirement for implementation of quantum search algorithm where no entanglement is generated throughout the process. For this purpose, we consider the Bernstein-Vazirani algorithm while we implement the oracle just once. We show while the output state remains a product one, the performance can be monotonically related to the l_1-norm of coherence of the initial state. Furthermore we generalize the protocol to higher dimension and show that quantum coherence plays the role of necessary and sufficient resource for the algorithm to give advantage over classical computation.

Introduction

it has been shown that to attain exponential speed-up in quantum algorithms with pure states, presence of multipartite entanglement becomes necessary. Even though entanglement play a crucial role in computational tasks with pure states. In 1997, Ethan Bernstein and Umesh Vazirani prescribed an algorithm (BV algorithm) where the job is to identify an unknown function encoded as a string in an oracle [1]. By performing this black-box operation just once, it is shown to be possible to find the encoded string in the quantum domain. Here we ask the question, what is the necessary resource requirement for BV algorithm to work perfectly and give an advantageous result over classical computers.

Coherence in Generalized BV Algorithm

We can generalize BV algorithm for a d-dimenssinal Hilbert space. We consider the following function:

$$\boldsymbol{f_a(x) = \frac{2\pi}{d} x.a}$$

By defining:

$$U_a = \sum_{x=1}^{d} e^{i f_a(x)} |x\rangle\langle x|$$

We have:

$$\boldsymbol{\langle \varphi_{max} | U_a^\dagger U_b | \varphi_{max} \rangle = \delta_{a,b}}$$

In which φ_{max} is a maximally l1 coherent [2] state. Therefore we can distinguish the function.
Again, if we perform the algorithm as the protocol explained in previous section we have the following relation:

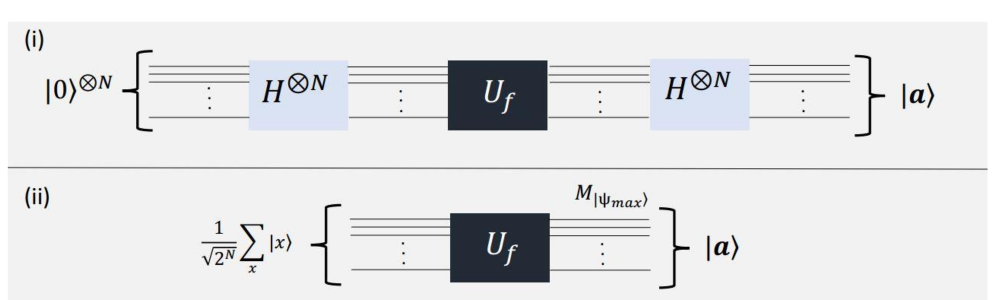
$$\boxed{\boldsymbol{F = \frac{1 + C_{l1}}{2}}}$$

Bernstein-Vazirani Algorithm

Using BV algorithm one tries to find an unknown N-bit string, say $\boldsymbol{a} = a_1 a_2 \ldots a_N$ encoded as a function $f(\boldsymbol{x}) = \boldsymbol{a.x} = \sum_i x_i a_i$ on the N-bit string **x**$= x_1 x_2 \ldots x_N$ in an oracle with only one query, as demonstrated in the figure (i) below.

If we use the initial state $\frac{|0\rangle + e^{i\varphi_k}|1\rangle}{\sqrt{2}}$ (φ_k is a phase.) for kth qubit , instead of applying the second Hadamard, we can measure the qubit in the basis $\left\{\frac{|0\rangle + e^{i\varphi_k}|1\rangle}{\sqrt{2}}, \frac{|0\rangle - e^{i\varphi_k}|1\rangle}{\sqrt{2}}\right\}$ as shown in figure (ii).

The states $\frac{|0\rangle + e^{i\varphi}|1\rangle}{\sqrt{2}}$ are Ideal states to initiate the algorithm with because they obtain $\boldsymbol{a}$ with the probability of 1.



Coherence in BV Algorithm

Let us now initiate the algorithm with the most general state . We perform as the following protocol:

First: we find the ideal state $\frac{|0\rangle + e^{i\varphi_k^*}|1\rangle}{\sqrt{2}}$ which has the maximum fidelity with the k-th qubit state in the initial state $\rho_{i,k}$.

Second: we apply the oracle unitary U_f:

$$\rho_{f,k} = U_f \rho_{i,k} U_f^\dagger$$

Finally: we compare the final k-th qubit state $\rho_{f,k}$ with $U_f \frac{|0\rangle + e^{i\varphi_k^*}|1\rangle}{\sqrt{2}}$ in terms of fidelity F.

We have the following relationship:

$$\boxed{\boldsymbol{\overline{F} = \frac{1 + \overline{C_{l1}}}{2}}}$$

in which $\overline{C_{l1}}$ $(\overline{F})$ is the average of C_{l1} [2](F) over all qubits.

Conclusion

In this work we try to explore the resource requirement in a quantum algorithm which shows potential speed up over the classical one where entanglement is not necessary. Also, we find a monotonic relationship between the fidelity and the l1-norm of coherence [2] for BV algorithm and its generalization for qdits.

References

[1] Bernstein, E., & Vazirani, U. (1997). Quantum complexity theory. *SIAM Journal on computing*, *26*(5), 1411-1473.

[2] Baumgratz, T., Cramer, M., & Plenio, M. B. (2014). Quantifying coherence. *Physical review letters*, *113*(14), 140401.