Rare events collision model for open quantum systems

DEPARTMENT OF PHYSICS

FQXi
FOUNDATIONAL QUESTIONS INSTITUTE

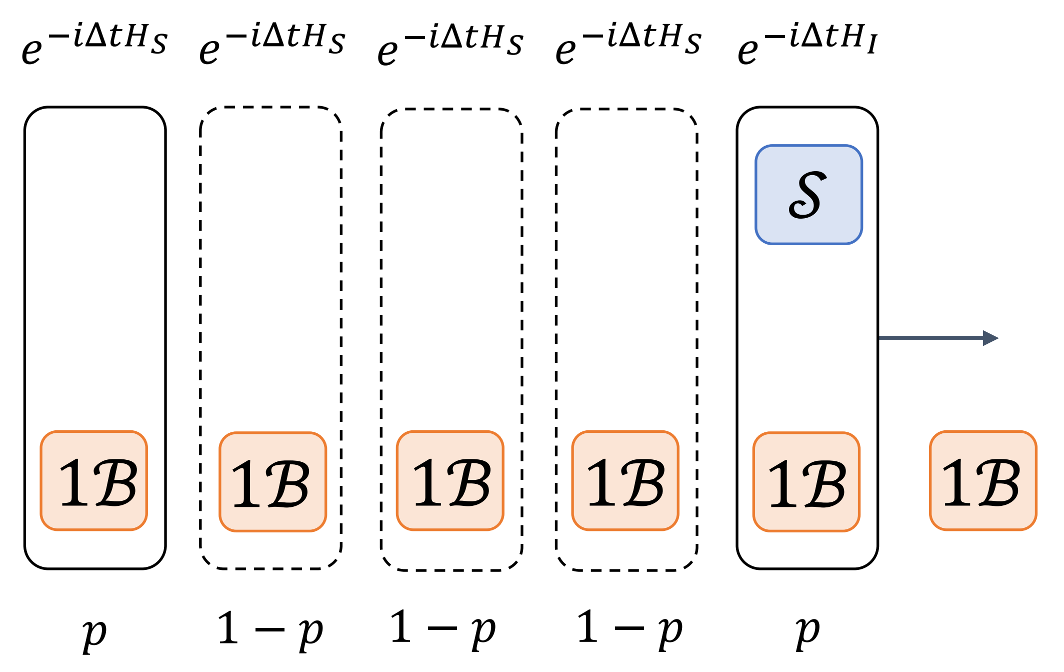
Guilherme de Sousa[1], Christopher Jarzynski[1]

[1]University of Maryland, College Park, Maryland, USA

Abstract

We study how one can use collision models[1] as a playground for quantum thermodynamics. A slight modification in the usual collision model is used to derive analytical master equations in the limit of rare interactions, that can be related to well known dilute gas models[2]. We established a close relation between these models and discrete state thermodynamics[3]. We hope to use such systems to explore stochastic thermodynamics from a quantum perspective.

Rare event model



$$\rho_s(t+\Delta t) = p\mathrm{Tr}_b[\mathcal{U}_C\rho_s(t)\otimes\sigma\mathcal{U}_C^\dagger] + (1-p)\mathcal{U}_F\rho_s(t)\mathcal{U}_F^\dagger$$

$$H = \frac{\omega_s}{2}\sigma_z\otimes\mathbb{I} + \frac{\omega_b}{2}\mathbb{I}\otimes\sigma_z + y(\sigma_x\otimes\sigma_x + \sigma_y\otimes\sigma_y)$$

In the limit of rare events, we can derive a CPTP map for the statistical ensemble of many copies of the system. If the interactions are fast, then a master equation holds. Note that rare collisions are effectively equivalent to weak coupling for a ME evolution.

These results were derived using a semiclassical approach without taking account the kinetic degrees of freedom. A careful derivation can be found in [2].

Effective Master Equation

Jump operators

$$\begin{aligned}\partial_t\rho &\\ = -i[H_s,\rho] &+ \Gamma_i\mathcal{D}[|1\rangle\langle 0|]\rho + \Gamma_o\mathcal{D}[|0\rangle\langle 1|]\rho\\ &+ \lambda_0\mathcal{D}[|0\rangle\langle 0|]\rho + \lambda_1\mathcal{D}[|1\rangle\langle 1|]\rho\end{aligned}$$

Measurement decoherence

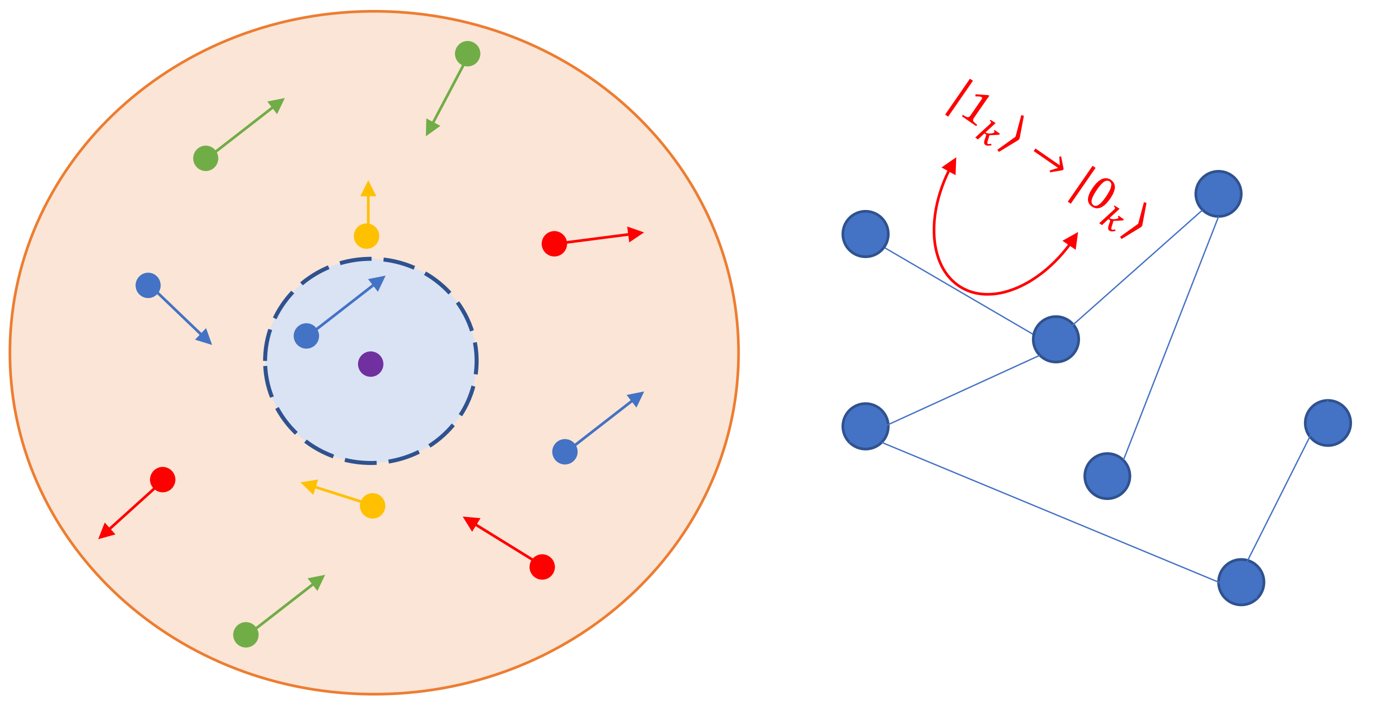
$$\Gamma_o = \chi\sin^2(y\Delta\mathrm{t})\cdot\sigma_{00}$$
$$\Gamma_i = \chi\sin^2(y\Delta\mathrm{t})\cdot\sigma_{11}$$
$$\Gamma_i = \Gamma_o e^{-\beta\omega_b}$$

$$\lambda_0 = \chi(1-\cos(y\Delta t))^2\cdot\sigma_{11}$$
$$\lambda_1 = \chi(1-\cos(y\Delta t))^2\cdot\sigma_{00}$$

This master equation has the property of thermal relaxation to the Gibbs state with respect to the bare Hamiltonian H_s.

$$\rho(t\to\infty) = \rho^{ss} = e^{\beta(F-H_s)}$$

We can have many "flavors" of gas particles that cause transitions in different states.



Thermodynamics of discrete state

$$\partial_t\rho = -i[H,\rho] + \sum\Gamma_{ij}\left(X_{ij}\rho X_{ij}^\dagger - \frac{1}{2}\{X_{ij}^\dagger X_{ij},\rho\}\right)$$
$$\Gamma_{ij} = \Gamma_{ji}e^{-\beta(\varepsilon_1-\varepsilon_2)},\qquad X_{ij} = |i\rangle\langle j|$$

$$\partial_t\rho_{ii} = \sum W_{i,j}\,\rho_{jj}$$
$$W_{ii} = -\sum W_{j,i}$$

$$\partial_t\rho_{i\neq j} = [-i\Delta\varepsilon_{ij} + 0.5(W_{ii}+W_{jj})]\rho_{ij}$$

$$\rho_{ii}^{eq} = e^{\beta(F-\varepsilon_i)}\qquad \rho_{i\neq j}^{eq} = 0$$

In this framework, any quantum initial state eventually relaxes to a classical state with no coherences in the energy basis. Quantum → Classical

Conclusion & References

- Collision models are tractable and very useful to derive ME
- Rare events are equivalent to weak coupling
- Thermodynamics of discrete states follows from Quantum dynamics

[1] S. Lorenzo, F. Ciccarello, and G.M. Palma. Phys. Rev. A **96**, 032107 (2017)

[2] S. N. Filippov, G. N. Semin, and A. N. Pechen, *Phys. Rev. A* ***101**, 012114 (2020)*

[3] C. Van den Broeck, M. Esposito, *Physica A* ***418** 6–16 (2015)*