Characterisation of multi-mode catalytic Gaussian thermal operations

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Abstract

Recently, it has been shown that a limited set of state transformation is realisable via Gaussian thermal operations, especially in the single-mode case. Here, we look at whether introducing catalysts can help us to achieve more state-transformation. We particularly consider two different catalytic scenarios: with or without correlation between the system and the catalyst at the end.

Gaussian Thermal Operations (GTO)

Gaussian thermal operations are the class of Gaussian operations on a continuous variable system obtained by energy-preserving interaction Hamiltonians between the system and a thermal bath.

Theorem (in [1]). *(Characterisation of GTOs) Let H_s be a system Hamiltonian with normal form $S^{-1}H_s(S^T)^{-1} = \bigoplus_l \omega_l \mathbb{I}_{2n_l}$ where n_l is the mode degeneracy of the eigenfrequency ω_l. The class of GTOs at background inverse temperature β act on the system covariance matrix σ as*

$$\sigma \xrightarrow{GTO} S(\bigoplus_l W_l \circ \Phi_l \circ Z_l[S^{-1}\sigma(S^T)^{-1}])S^T,$$

where (i) W_l and Z_l are passive linear transformations, and (ii) Φ_l is a CP maps realisable by adding an ancilla bath mode, applying a passive linear transformation to the system and the bath modes, and tracing out the bath mode.

The authors in [1] also fully characterised single-mode GTOs; derived both necessary and sufficient conditions for state transformation under single-mode GTOs.

The above theorem implies that non-trivial state transformations happen within **degenerate modes**. What kind of thermodynamically interesting transformations is allowed in multi-mode degenerate systems?

Catalytic Gaussian Thermal Operations

As a special case of multi-mode GTOs, we look at catalytic Gaussian thermal operations. We consider two different catalytic scenarios; ***strong catalysts*** and ***weak catalysts***.

1. **Strong catalysts**: the catalytic mode needs to be uncorrelated from the system at the end.
$$\rho_S \otimes \rho_C \xrightarrow{GTO} \rho_S' \otimes \rho_C$$
2. **Weak catalysts**: the catalytic mode can be correlated to the system at the end.
$$\rho_S \otimes \rho_C \xrightarrow{GTO} \rho_{SC}' \quad with \quad tr_S[\rho_{SC}'] = \rho_C$$

The main question

Do such catalytic transformations allow for more possible operations to be performed on a system?

[1] A. Serafini *et al.*, Phys. Rev. Lett. 124, 010602 (2020)
[2] R. Simon *et al.*, Phys. Rev. A 49, 1567 (1994).

Technical ingredient: Normal form

In this work, we largely make use of normal form [2], which is a useful decomposition of the covariance matrix. We decompose the 2n x 2n covariance matrix into two n x n matrices:

$$A_{ij} \coloneqq \langle a_j^\dagger a_i \rangle - \nu, \qquad B_{ij} \coloneqq \langle a_j a_i \rangle,$$

where ν is the variance of any quadrature in a thermal state at background inverse temperature β. We can recover the covariance matrix from

$$\langle x_i x_j \rangle = Re[A_{ij} + B_{ij}] + \delta_{ij}\nu, \qquad \langle p_i p_j \rangle = Re[A_{ij} - B_{ij}] + \delta_{ij}\nu$$
$$\frac{1}{2}\langle \{x_i, p_j\} \rangle = Im[B_{ij} - A_{ij}].$$

We characterize state in terms of eigenvalues $\{\alpha_i\}_i$ of the matrix A and the singular values $\{\beta_i\}_i$ of the matrix B. The two matrices are in general not simultaneously diagonalizable, so they do not in general provide a full characterization of covariance matrices.

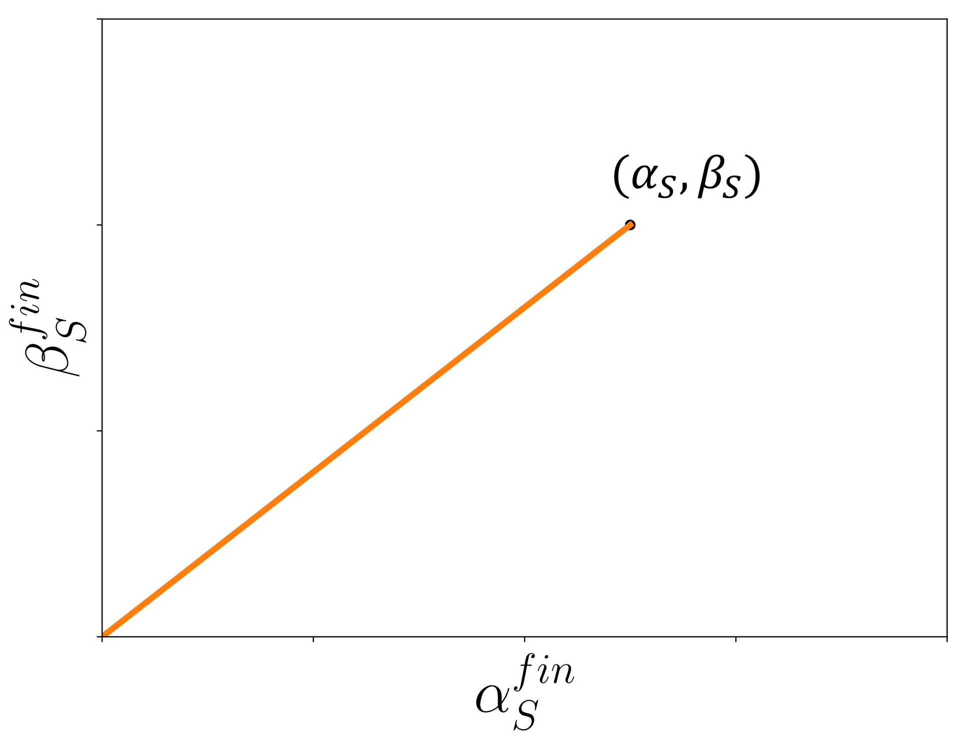
Single-mode case: a single-mode system and a single-mode catalyst

When both the system and the catalyst are single-mode, the initial A and B matrices are simultaneously diagonalizable, since the system and the catalyst must be uncorrelated in the beginning. Thus, it is possible to fully characterize state transformations under single-mode catalytic GTOs with $\{\alpha_i\}_i$ and $\{\beta_i\}_i$.

Strong catalysts

For a given single-mode system characterized by (α_S, β_S), the final state with $(\alpha_S^{fin}, \beta_S^{fin})$ satisfies

$$\beta_S^{fin} \geq 0, \qquad |\alpha_S^{fin}| \leq |\alpha_S|,$$
$$\frac{\beta_S^{fin}}{\alpha_S^{fin}} = \frac{\beta_S}{\alpha_S}$$



Weak catalysts

For a given single-mode system characterized by (α_S, β_S), the final state with $(\alpha_S^{fin}, \beta_S^{fin})$ satisfies

$$\beta_S^{fin} \geq 0, \qquad |\alpha_S^{fin}| \leq |\alpha_S|,$$
$$\frac{\beta_S^{fin}}{\alpha_S^{fin}} \leq \frac{\beta_S}{\alpha_S}$$