Two-temperature estimation in the quantum regime: using Mach-Zehnder interferometer and quantum process framework

Harshit Verma*, Carolyn E. Wood,
Magdalena Zych, Fabio Costa

Australian Centre of Excellence for engineered quantum systems (EQUS), School of Mathematics and Physics, University of Queensland, St Lucia, QLD 4072, Australia

* h.verma@uq.edu.au

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA
CREATE CHANGE

EQUS
Australian Research Council
Centre of Excellence for
Engineered Quantum Systems

Multiparameter estimation & CR bounds

State preparation of qubit(s) as probe(s)

Temperature(s) encoded through interaction with bath(s)

Measurement and estimation of temperature

Multiparameter CR bound :

$$\boxed{\mathrm{Cov}(\vec{T}) \geq \frac{Q_T^{-1}}{\nu}}$$

$$Q_T = \begin{pmatrix} Q_{T_1T_1} & Q_{T_1T_2} \\ Q_{T_1T_2} & Q_{T_2T_2} \end{pmatrix}$$: Quantum Fisher Info. Matrix (QFIM)

Inv(QFIM) : $$\mathcal{Q}_T^{-1} = \frac{1}{\det(\mathcal{Q}_T)} \begin{pmatrix} \mathcal{Q}_{T_2T_2} & -\mathcal{Q}_{T_1T_2} \\ -\mathcal{Q}_{T_1T_2} & \mathcal{Q}_{T_1T_1} \end{pmatrix}$$

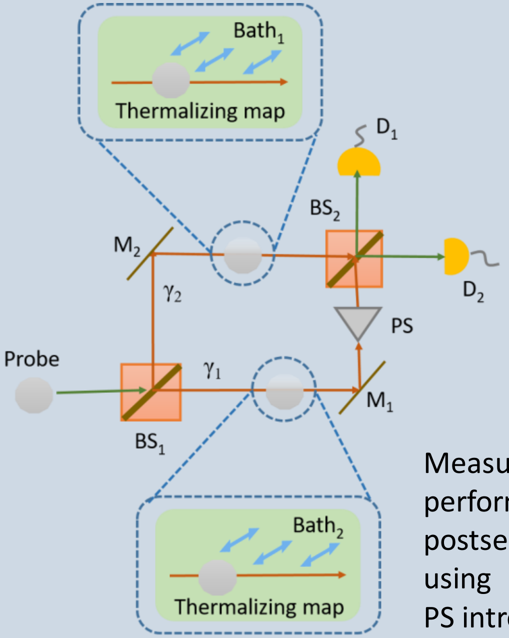
Bounds :

$$\mathrm{Var}(T_1) \geq \frac{\mathcal{Q}_{T_2T_2}}{\nu \det(\mathcal{Q}_T)}$$

$$\mathrm{Var}(T_2) \geq \frac{\mathcal{Q}_{T_1T_1}}{\nu \det(\mathcal{Q}_T)}$$

$$\left(\mathrm{Var}(T_2) - \frac{\mathcal{Q}_{T_1T_1}}{\nu \det(\mathcal{Q}_T)}\right) \geq \left[\mathrm{Cov}(T_1, T_2) + \frac{\mathcal{Q}_{T_1T_2}}{\nu \det(\mathcal{Q}_T)}\right]^2$$

Quantum Processes & MZ setup

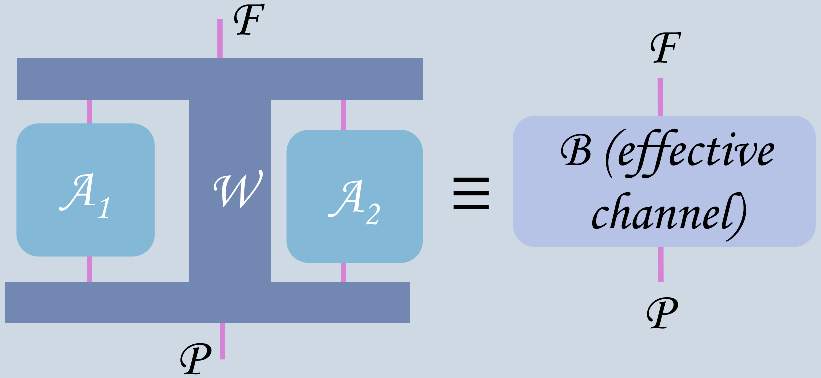


MZ setup:

- 1-bath: The state of the (thermal) bath depends on the control (path).
- 2-bath: Two independent baths on the path of the MZ.

Measurement, estimation performed using probe states postselected on path DOF using detectors D_1 and D_2. PS introduces a phase: Φ.

Process: A supermap from the set of linear operations (CP maps) on an input Hilbert space to the set of linear operations on an output Hilbert space. Example:



$$\boxed{\mathcal{J}_{\mathcal{B}} = \mathrm{Tr}_{(i=1,2,...,k)}\left(\mathbb{W}^T \mathcal{J}_i \otimes^{(i=1,2,...,k)} \otimes \mathbb{I}_{\mathcal{B}}\right)}$$

Bath(s) and their interaction with probe

Purified thermal bath constituted by 2 qubits, written in the energy eigenbasis as follows, interaction through unitary (U):

$$|\theta^\beta\rangle \equiv \frac{e^{-E_0\beta/2}}{\sqrt{e^{-E_0\beta} + e^{-E_1\beta}}}|0,0\rangle + \frac{e^{-E_1\beta/2}}{\sqrt{e^{-E_0\beta} + e^{-E_1\beta}}}|1,1\rangle$$

$$T = 1/\beta$$

$$H_B = H_0 \otimes \mathbb{I}_2 + \mathbb{I}_2 \otimes H_0$$
$$H_0 = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

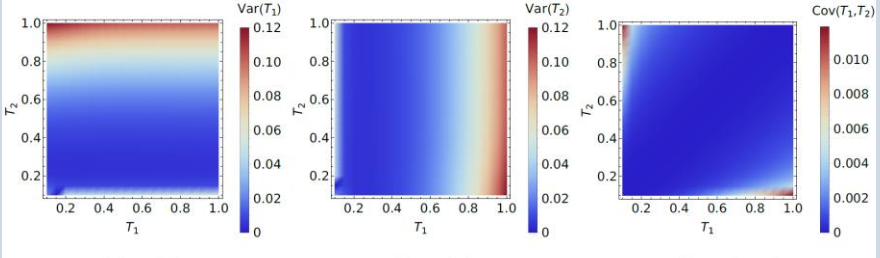
$$\boxed{U_{PB}^{\eta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-\eta} & \sqrt{\eta} & 0 \\ 0 & -\sqrt{\eta} & \sqrt{1-\eta} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

Results using MZ setup

For a single qubit as a probe:

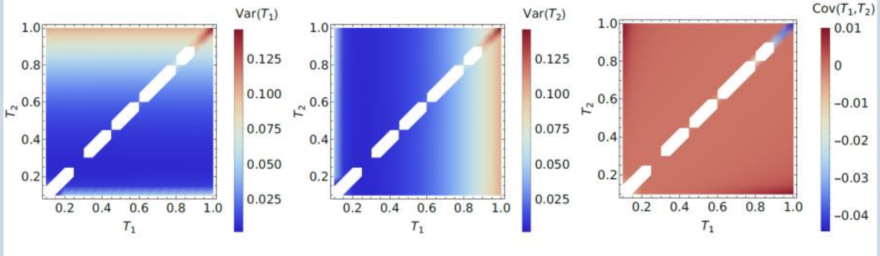
- 1-bath/ 2-bath : |QFIM| = 0 ⇒ Bounds cannot be obtained.

For a two- qubit probe, |QFIM| ≠ 0 (generally)



(a) Var(T_1) (b) Var(T_2) (c) Cov(T_1, T_2)

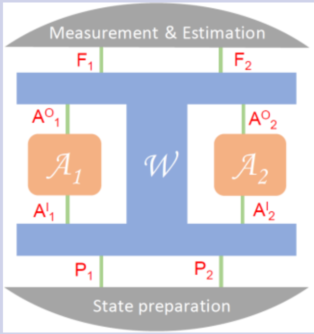
1 – bath ($\Phi = \pi/2$), $\nu = 100$



(a) Var(T_1) (b) Var(T_2) (c) Cov(T_1, T_2)

2 – bath ($\Phi = \pi/2$), $\nu = 100$

Results using Switch & Det processes



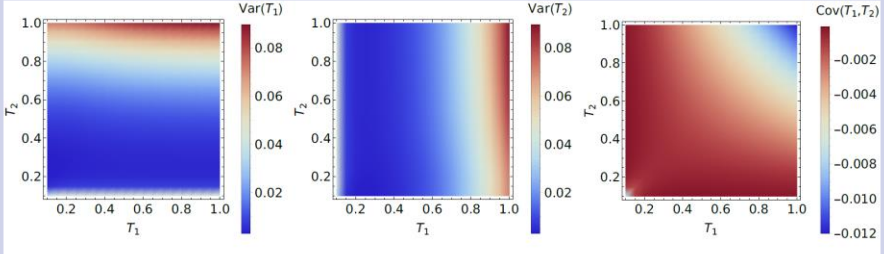
v = 100

$$\rho_{\text{in}} = |0\rangle\langle 0| \otimes |+\rangle\langle +|$$

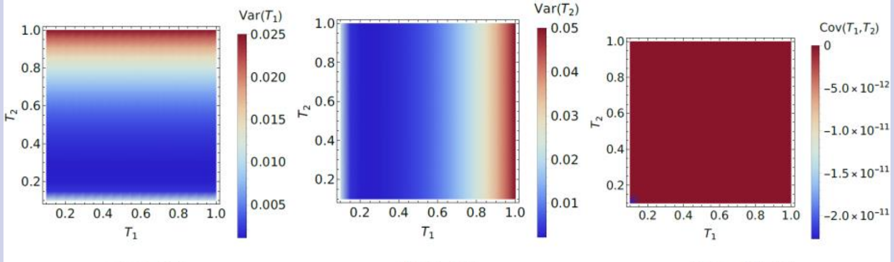
$$\mathbb{W}_{\text{switch}} = |w_{\text{switch}}\rangle\langle w_{\text{switch}}|$$

$$|w_{\text{switch}}\rangle = |0\rangle^{P_1}|\mathbf{1}\rangle\rangle^{P_2 A^I_1}|\mathbf{1}\rangle\rangle^{A^O_1 A^I_2}|\mathbf{1}\rangle\rangle^{A^O_2 F_2}|0\rangle^{F_1} + |1\rangle^{P_1}|\mathbf{1}\rangle\rangle^{P_2 A^I_2}|\mathbf{1}\rangle\rangle^{A^O_2 A^I_1}|\mathbf{1}\rangle\rangle^{A^O_1 F_2}|1\rangle^{F_1}$$

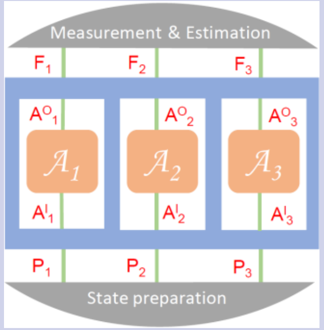
- Bi-partite process
- Conforms to indefiniteness in causal order.
- Doesn't violate causal inequality.
- Measurement & estimation performed on $F_1 \otimes F_2$
- Can be used to measure two temperatures simultaneously



(a) $\text{Var}(T_1)$ (b) $\text{Var}(T_2)$ (c) $\text{Cov}(T_1,T_2)$



(a) $\text{Var}(T_1)$ (b) $\text{Var}(T_2)$ (c) $\text{Cov}(T_1,T_2)$



- Tri-partite process
- Violates causal inequality.
- Measurement & estimation performed on $F_1 \otimes F_2 \otimes F_3$
- Can be used to measure two temperatures simultaneously.
- The variance depends on how many instances of thermalizing maps of a particular temperature are used.

$$\rho_{\text{in}} = |\psi_0\rangle\langle\psi_0| \otimes |+\rangle\langle +| \otimes |+\rangle\langle +|$$

v = 100

$$\mathbb{W}_{\text{Det}} = |w_{\det}\rangle\langle w_{\det}|$$

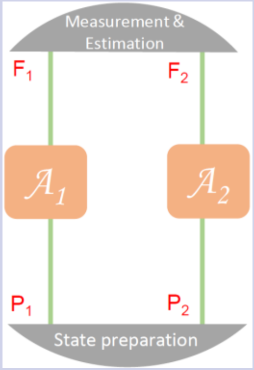
$$|w_{\det}\rangle = \sum_{i,j,k,r,s,t} |r \oplus \neg j \wedge k, s \oplus \neg k \wedge i, t \oplus \neg i \wedge j\rangle \otimes |r,s,t\rangle \otimes |i,j,k\rangle \otimes |i,j,k\rangle$$

$\neg$: NOT $\wedge$: AND

Conclusions

Apparatus	No. of Probe qubits	| QFIM |	Attainable	Lower Limit of variance	Upper Limit of variance	Comparison with parallel
1-bath MZ	1	$= 0$	N.A.	–	–	–
2-bath MZ	1	$= 0$	N.A.	–	–	–
1-bath MZ	2	$\neq 0$	Yes	0	0.12	🟥
2-bath MZ	2	$\neq 0$	Yes	0	0.13	🟥
Bipartite quantum switch	2	$\neq 0$	Yes	0	0.08	🟦
Det Process	3	$\neq 0$	Yes	0	0.05	🟦
Parallel scheme	2	N.A.	Yes	0	0.05	🟩
2 X Bipartite quantum switch	4	N.A.	Yes	0	0.04	🟦
2 X Det Process	6	N.A.	Yes	0	0.02	🟦

TABLE I: Summary of the results on the simultaneous estimation of two temperatures. A qualitative comparison between the various setups – keeping in mind the variance limits and the number of qubits used as a probe is also presented. The qualitative results are bench-marked against the parallel (independent) two temperature estimation scheme (🟩) with 🟦 representing a reasonably close quality (good), and 🟥 representing a worse quality. N.A. = Not Applicable.



Parallel scheme with simultaneous measurement of two temperatures using two independent/uncorrelated probes outperforms all the others.

Ongoing & Future work

- Although no significant advantage vis-à-vis variances obtained, the diagonalization of the covariance matrix can lead to interesting results.
- For example, we can measure certain linear combinations of temperatures: not possible using the parallel scheme.
- Shown to be true in Quantum Switch and MZ setups.
- For future: using Bayesian inference method in the quantum process framework for thermometry.

References

Araújo, M., Feix, A., Navascués, M., & Brukner, C. *A purification postulate for quantum mechanics with indefinite causal order*. *Quantum, 1*, 1–13 (2017).

Oi, D. K. L. *Interference of Quantum channels. Phys. Rev. Lett.*, *91*(6), 67902 (2003).

Liu, J., Yuan, H., Lu, X.-M., & Wang, X. *Quantum Fisher Information Matrix and Multiparameter Estimation. Journal of Physics A: Mathematical and Theoretical*, *53*(2), 23001 (2019).

Mukhopadhyay, C., Gupta, M. K., & Pati, A. K., *Superposition of causal order as a metrological resource for quantum thermometry. arXiv:1812.07508v1 (2018).*