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D PHYS

Online Erasure in Quantum Computers

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Motivation

Presently, quantum computers rely on low temperatures to preserve coherence, and their memory is limited. Most current quantum algorithms are implemented through unitary circuits, and some registers are measured at the end. Eventually all registers must be erased (that is, reset to $|0\rangle$), so they are ready for the next computation. A naïve erasure of these registers costs work and dissipates heat to the quantum computer's environment, due to Landauer's principle [4]. To avoid this, there are proposals for efficient erasure of quantum registers [1, 8, Fig. 8]; these usually involve reversing parts of the original circuit. Here we go one step further and investigate whether there is an advantage in making use of the entanglement between different registers in the middle of a quantum algorithm to erase ancillas as they stop being useful — we call this *online erasure*.

Erasure with a quantum memory

This idea is an application of [6], which generalizes Landauer's principle: if we want to erase a quantum register or system S, while preserving a quantum memory M, by acting on SM and a thermal environment at temperature T, that is $\rho_{SM} \xrightarrow{\mathcal{E}} |0\rangle\langle 0|_S \otimes \rho_M$, then an optimal process $\mathcal{E}$ will on average dissipate heat

$$Q = H(S|M) k_B T \ln 2$$

to the environment, where k_B is the Boltzmann constant, and $H(S|M)$ the conditional von Neumann entropy.

Model: battery and computation registers

We take a simplified model for a quantum computer whose memory is split into two or registers: the computation zone, where algorithms are implemented, and a battery zone that stores erased qubits in state $|0\rangle$ (the "good" qubits) and some mixed qubits ("bad" qubits). The idea is to shift the thermodynamic process of erasure to the battery: there, fully mixed qubits can be brought to a pure state at a fixed work/heat cost, for example through the procedure in [7]. To erase a qubit in the computation zone, we simply swap it with a good one from the battery, and the work cost of a computation can be quantified by the number of good qubits from the battery used (Fig. 1).

Unmodified HSP Algorithm

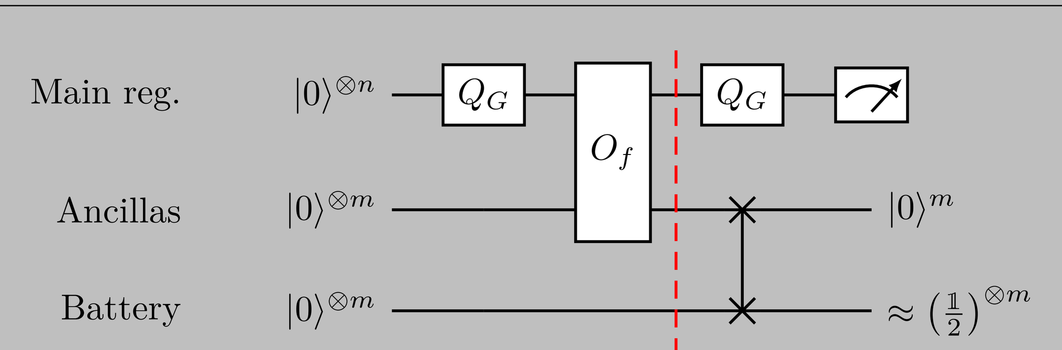

Figure 1. Here, Q_G is the quantum Fourier transform for group G and O_f is the function oracle, $O_f|x\rangle|y\rangle = |x\rangle|f(x) \oplus y\rangle$. Naïvely, one would erase the ancillas by swapping them with the battery register. We have $n = \log_2|G|$, $m = \log_2|S|$ qubits for G and S respectively. The dashed line indicates the position of online erasure in the modified versions ahead.

Algorithm

We focus on erasing the ancillary output register S of the algorithm after the oracle is applied (dashed line in Fig. 1). We want to preserve the state of the "memory" input register G (named after the input group). Their joint state at this point is denoted by ρ_{GS}. First, local unitaries U_G and U_S compress the correlations between the two registers into Bell pairs, bringing the state to a product of Bell pairs between subsystems of S and G and some leftover state:

$$\rho_{GS} \xrightarrow{U_G \otimes U_S} \tilde{\rho}_{G_1 S_1} \otimes \left(|\chi\rangle\langle\chi|^{\otimes \ell}\right)_{G_2 S_2},$$

where $G = G_1 \otimes G_2$, $S = S_1 \otimes S_2$, each Bell pair $|\chi\rangle$ is formed by a qubit from G_2 and one from S_2, and $\ell = \log_2|G_2| = \log_2|S_2|$. These ℓ Bell pairs are swapped with fully mixed states $(\mathbb{1}/2)^{\otimes 2\ell}$ from the battery (i.e. "bad qubits"). This reduces the entropy of the battery by 2ℓ. Then, unitary U_G on G is undone, leaving the reduced state of G unchanged, respecting our memory preservation condition, so algorithm still solves the HSP. Finally, S is erased by swapping it with good qubits from the battery (Fig. 2).

Modified Protocol HSP Algorithm.

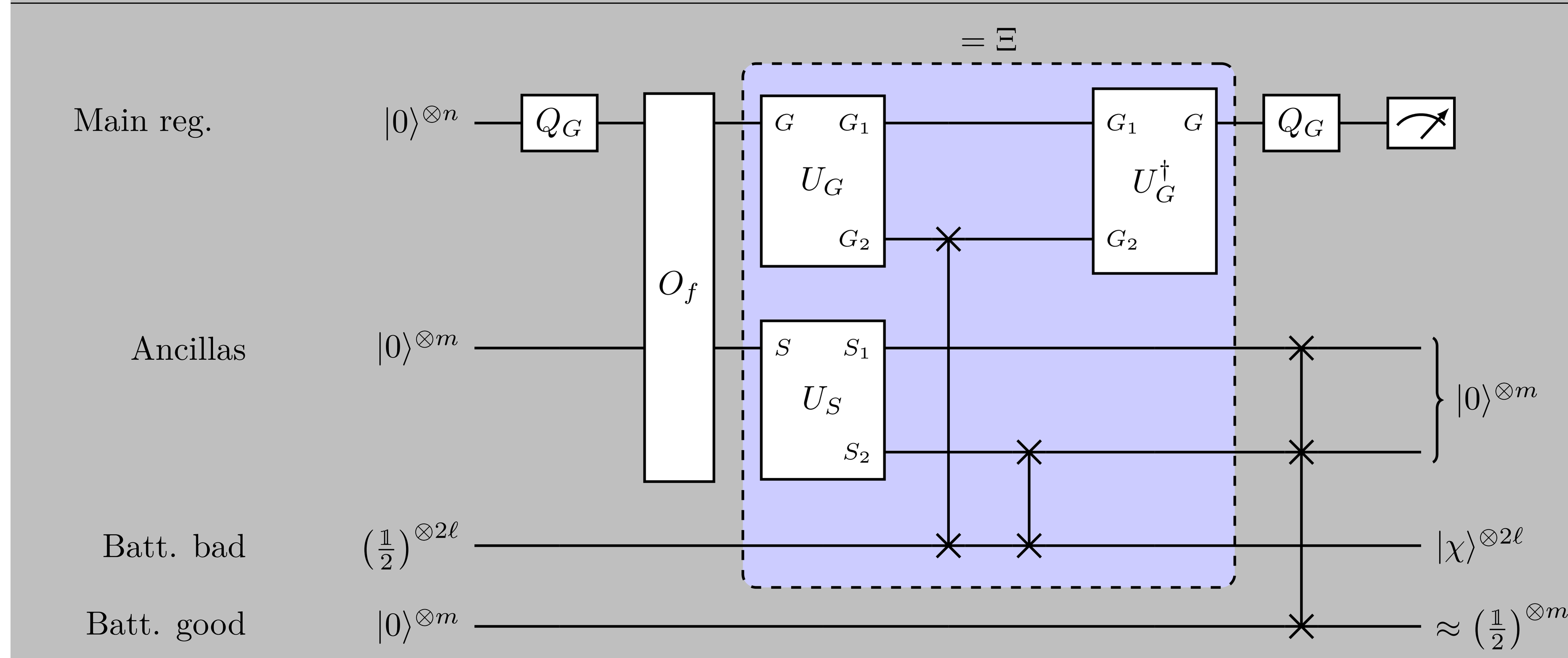

Figure 2. The last two registers are batteries ("bad" and "good"). The map Ξ (violet box) swaps the entangled qubits from $G_2 \otimes S_2$ with mixed states from the first battery.

Cost analysis

The naïve procedure would have cost $\log_2|S|$ clean qubits from the battery; in contrast, the overall cost of our proposed procedure is $\log_2|S| - 2\ell$ clean qubits. Here ℓ is in principle upper-bounded by the conditional entropy $|H(S|G)| = \log_2|G/H| =: \ell_{\max}$. However, the number of Bell pairs we can actually extract depends on our information about the function f (or alternatively some properties of the hidden subgroup H). Indeed, we can show that finding the unitaries U_S and U_G that extract $\ell_{\max}$ Bell pairs corresponds to solving the HSP itself. Therefore, the cases of interest are those with $0 < \ell < \ell_{\max}$, where we can find appropriate unitaries if we have partial information on H. For example, one could know an intermediate subgroup, $H \subseteq K \subseteq G$, giving us the factorization $G = (G/K) \otimes K$.

Oracle simplification

We propose an alternative algorithm for the cases when we have open circuit access to the oracle (as opposed to black box access). This holds, for example, for order finding, a simple special case of the HSP. From the previous modifications we have a unitary U_G that factors $G = G/K \otimes K$, together with a unitary U_S for factoring S. Reordering these operations around the oracle according to Fig. 3 gives a modified oracle $\tilde{O}_f$ which can solve the HSP with 2ℓ variable qubits less than O_f.

Simplification of the HSP Algorithm

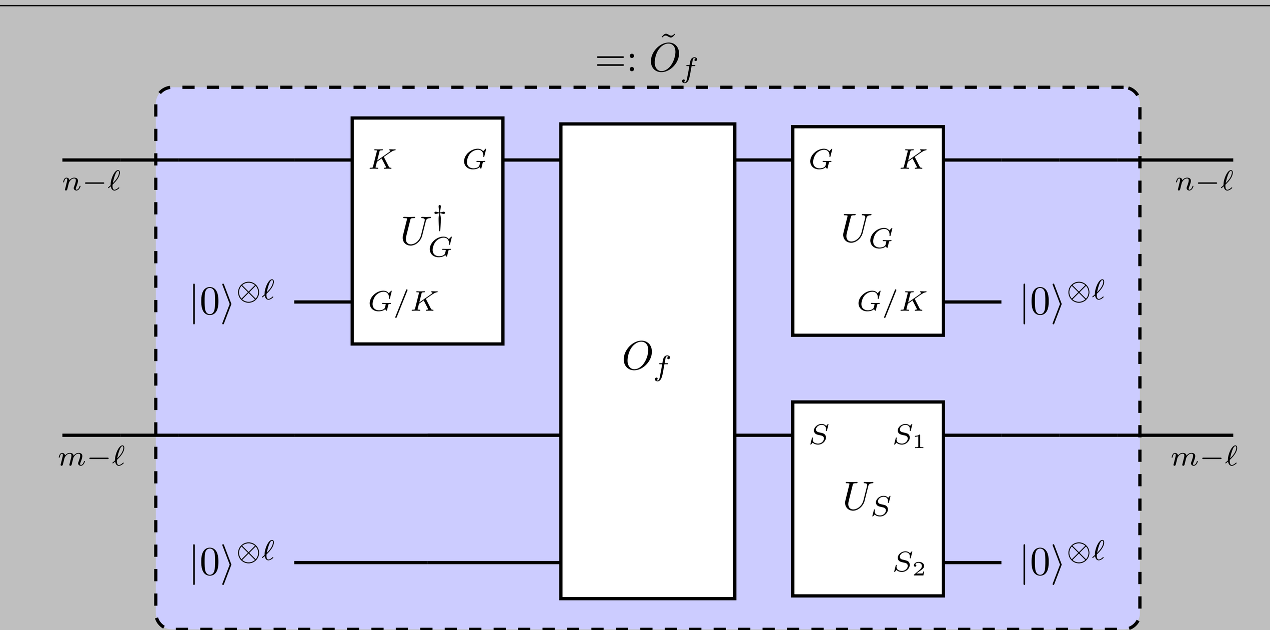

Figure 3. The modified oracle $\tilde{O}_f$ replaces O_f. The lowercase letters indicate the number of qubits.

References

[1] Charles H. Bennett. The thermodynamics of computation—a review. *International Journal of Theoretical Physics*, 21(12):905–940, December 1982.

[2] Ronald de Wolf. Quantum computing: Lecture notes, 2019.

[3] Alexei Yurievich Kitaev. Quantum measurements and the abelian stabilizer problem, 1995.

[4] R. Landauer. Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development*, 5(3):183–191, 1961.

[5] Michele Mosca and Artur Ekert. The hidden subgroup problem and eigenvalue estimation on a quantum computer. 3 1999.

[6] Lídia del Rio, Johan Åberg, Renato Renner, Oscar Dahlsten, and Vlatko Vedral. The thermodynamic meaning of negative entropy. *Nature*, 474(7349):61–63, Jun 2011.

[7] Paul Skrzypczyk, Anthony J. Short, and Sandu Popescu. Work extraction and thermodynamics for individual quantum systems. *Nature Communications*, 5(1), June 2014.

[8] John Watrous. Quantum computational complexity, 2008.