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Quantum Heat Engines with Carnot Efficiency at Maximum Power

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Conventional heat engines, be these classical or quantum, with higher power yield lesser efficiency and vice versa and respect various power-efficiency trade-off relations. Here we show that these relations are not fundamental. We introduce quantum heat engines that deliver maximum power with Carnot efficiency in the one-shot finite-size regime. These engines are composed of working systems with a finite number of quantum particles and are restricted to one-shot measurements. The engines operate in a one-step cycle by letting the working system simultaneously interact with hot and cold baths via semi-local thermal operations. By allowing quantum entanglement between its constituents and, thereby, a coherent transfer of heat from hot to cold baths, the engine implements the fastest possible reversible state transformation in each cycle, resulting in maximum power and Carnot efficiency. We propose a physically realizable engine using quantum optical systems.

SEMI-LOCAL THERMAL OPERATION

The semi-local thermal operation [1] is the special type of thermal operation(global). That leads the global unitary operation is such way that, subsystem S_1 is locally interacting with hot bath B_1 and subsystem S_2 is locally interacting with cold bath B_2,

$$\Lambda_{S_{12}}(\rho_{S_{12}}) := \mathrm{Tr}_{B_1B_2}[U\,(\gamma_{B_1}\otimes\gamma_{B_2}\otimes\rho_{S_{12}})\,U^\dagger],$$

$$[U,\, H_{B_1}+H_{B_2}+H_{S_1}+H_{S_2}]=0, \quad \Leftarrow \quad \text{Total eneregy conservation}$$

$$[U,\, \beta_1(H_{B_1}+H_{S_1})+\beta_2(H_{B_2}+H_{S_2})]=0 \quad \Leftarrow \quad \text{Weighted energy conservation}$$

2ND LAWS FOR BLOCK DIAGONAL STATE

Under semi-local thermal operation, the block diagonal $[\rho_{S_{12}}, H_{S_{12}}]=0$, state transformation $(\rho_{S_{12}}, H_{S_{12}}) \to (\sigma'_{S_{12}}, H'_{S_{12}})$ is possible if, and only if,

$$S_\alpha(\rho_{S_{12}},\, \gamma_{S_1}\otimes\gamma_{S_2}) \geqslant S_\alpha(\sigma'_{S_{12}},\, \gamma'_{S_1}\otimes\gamma'_{S_2}), \quad \forall\, \alpha\geqslant 0,$$

$$S_\alpha(\rho_{S_{12}},\gamma_{S_1}\otimes\gamma_{S_2}) = D_\alpha(\rho_{S_{12}}\,\|\,\gamma_{S_1}\otimes\gamma_{S_2}) - \ln Z_1Z_2, \quad \forall\alpha\in[-\infty,\infty],$$

Renyi realtive entropy, $D_\alpha(\rho\,\|\,\gamma) = \dfrac{\mathrm{sgn}(\alpha)}{\alpha-1}\ln \mathrm{Tr}[\rho^\alpha\gamma^{1-\alpha}], \quad \forall\ \alpha\in[-\infty,\infty]$.

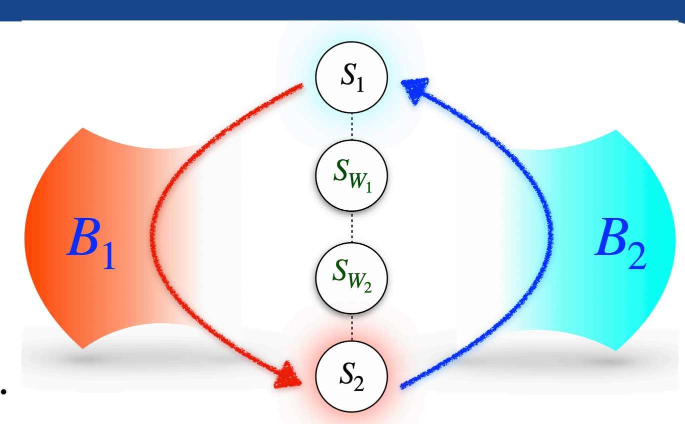
HEAT ENGINE WITH ONE-STEP CYCLE

$$(\rho_{S_{12}}, H_{S_{12}}) \;\to\; (\sigma'_{S_{12}}, H'_{S_{12}}),$$

where $H_{S_{12}} = H_{S_1}+H_{S_2}$, $H'_{S_{12}} = H'_{S_1}+H'_{S_2}$,

staisfy the conditions

$$\sigma'_{S_{12}} = U^{\mathrm{swap}}_{S_1\leftrightarrow S_2}(\rho_{S_{12}}),\ \ H'_{S_1}=H_{S_2},\ \ \text{and}\ \ H'_{S_2}=H_{S_1}.$$



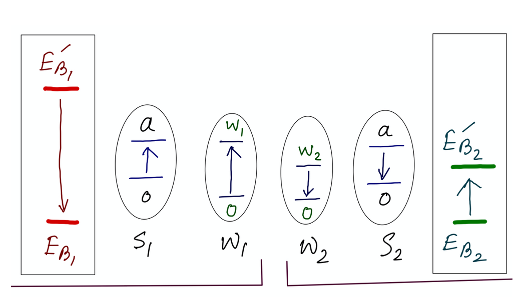
ATTAINING CARNOT EFFICIEY

Consider, subsystems are pure states and undergo state transformation via semi-local thermal operation

$$\rho_{S_{12}} = |0\rangle\langle 0|\otimes|1\rangle\langle 1| \;\to\; \sigma_{S_{12}} = |0\rangle\langle 0|\otimes|1\rangle\langle 1|$$

Weighted energy conservation

$$\beta_1E_{B_1}+\beta_2(E_{B_2}+a) = \beta_1(E'_{B_1}+a+W_1)+\beta_2(E'_{B_2}+W_2).$$



The free entropic different initial and final state

$$S_\alpha(\rho_{S_{12}}) \;-\; S_\alpha(\sigma_{S_{12}}) = a\beta_1 \;-\; a\beta_2 = \beta_1W_1+\beta_2W_2$$

$$\beta_1(E'_{B_1}-E_{B_1}) \;+\beta_2(E'_{B_2}-E_{B_2}) = \beta_1Q_1+\beta_2Q_2 = 0 \quad \Leftarrow \quad \text{Clausious equality}$$

Total energy conservation

$$E_1 \;-\; E'_1 \;+\; E_2 \;-\; E'_2 \;=\; W_1+W_2 \;=\; Q_1 \;+\; Q_2 \;=\; W_{ext}$$

$$\eta_C \;=\; \frac{W_{ext}}{Q_1} \;=\; 1 \;-\; \frac{Q_1}{Q_2} \;=\; 1 \;-\; \frac{\beta_1}{\beta_2} \quad \Leftarrow \text{The exact Carnot efficiency [1].}$$

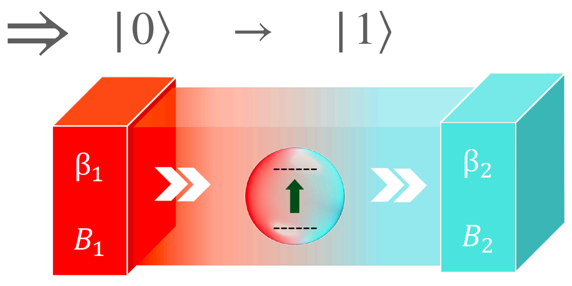
MAXIMUM POWER WITH CARNOT EFFICIENCY

State of the systems and batteries in compact form

$$|0,\,1,\,0,\,0\rangle_{S_1S_2S_{W_1}S_{W_2}} \to |1,\,0,\,1,\,1\rangle_{S_1S_2S_{W_1}S_{W_2}} \quad \Rightarrow \quad |0\rangle \quad \to \quad |1\rangle$$

Now the overall transformation becomes

$$\gamma_{B_1}\otimes\gamma_{B_2}\otimes|0\rangle\langle 0|_S \xrightarrow{U} \tau_{B_1B_2}\otimes|1\rangle\langle 1|_S$$



$$\Big[U,\, H_{B_1}+H_{B_2}+H_S\Big]=0 \quad \text{and} \quad \Big[U,\, \beta_1H_{B_1}+\beta_2H_{B_2}\Big]=0.$$

A generic interaction Hamiltonian

$$H_{in} = \hbar g \bigoplus_{E_{B_{12}}+a_0}\;\sum_{i=1}^{d_1(E_{B_1})d_2(E_{B_2})} |E'_{B_{12}}(i),\,1\rangle\langle E_{B_{12}}(i),\,0|_{B_1B_2S} + h.c.$$

The time evolved global state is an **entangled** state

$$|\psi(t)\rangle = U(t)\,|E_{B_1}(i),\,E_{B_2}(i),\,0\rangle_{B_1B_2S}$$

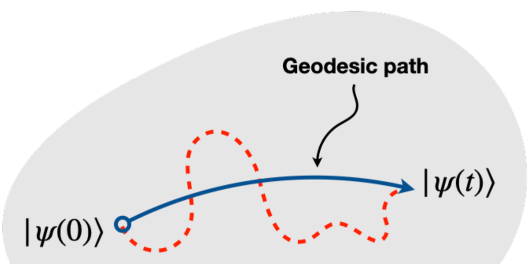
$$= \cos(gt)\;|E_{B_1}(i),\,E_{B_2}(i),\,0\rangle_{B_1B_2S} - i\sin(gt)\;|E'_{B_1}(i),\,E'_{B_2}(i),\,1\rangle_{B_1B_2S}$$

Enables coherent heat transfer!!

The speed of quantum evolution [2], $\quad v=\dfrac{ds}{dt}=\dfrac{\Delta H_{in}}{\hbar}=g$

$$ds = \frac{1}{2}(1-|\langle\psi(t)|\psi(t+dt)\rangle|^2)$$

$$\Delta H_{in} = \sqrt{\langle\psi(t)|H_{in}^2|\psi(t)\rangle - \langle\psi(t)|H_{in}|\psi(t)\rangle^2}$$



Delivers maximum power [3], $\quad P=\dfrac{W_{ext}}{\tau}=\dfrac{2gW_{ext}}{\pi}$

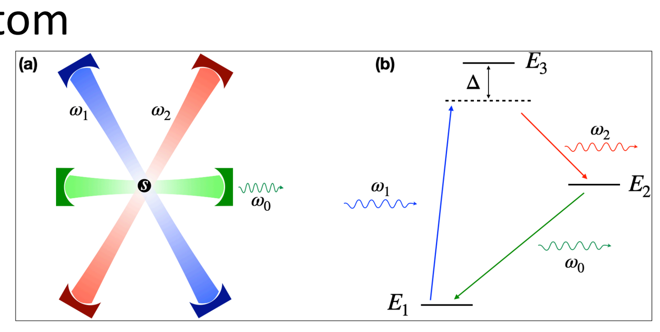
A QUANTUM OPTICS BASED HEAT ENGINE

Two baths at two temperatures and a 3-level atom

$$H_0 = H_{B_1}+H_{B_2}+H_S$$

$$H_{B_1} = \hbar\omega_1a_1^\dagger a_1;\;\; H_{B_2} = \hbar\omega_2a_2^\dagger a_2;$$

$$H_S = \sum_{i=1}^{3}E_i|i\rangle\langle i|_S \;\;\text{with}\;\; E_1=0.$$



Interaction Hamiltonian, with intensity-dependent couplings

$$H_I = f_1(N_1)+f_2(N_2)+\hbar g_1\;\theta_1(N_1)\;(a_1\sigma_{31}+h.c.)+\hbar g_2\;\theta_2(N_2)\;(a_2\sigma_{32}+h.c.)$$

After rotating-wave approximation

$$H'_I = \hbar g(A_1A_2^\dagger\sigma_{12}+A_1^\dagger A_2\sigma_{21});\quad A_k=a_kN_k^{-1/2};\;\; N_k=a_k^\dagger a_k;\qquad g=g_1g_2/\Delta;$$

$$H'_S = \frac{1}{2}\hbar\omega_0\;(|2\rangle\langle 2|_S-|1\rangle\langle 1|_S);\quad \omega_0\approx\omega_1-\omega_2;\qquad \Delta=(E_3-E_k)/\hbar-\omega_k$$

The corresponding unitary $\quad U(t)=\exp[-itH'_I/\hbar]$

$$[U(t),\,H'_S+H_{B_1}+H_{B_2}]=0 \;\;\text{and}\;\; [U(t),\,\beta_1H_{B_1}+\beta_2H_{B_2}]=0.$$

Work extracted per engine cycle $\hbar\omega_0$

Time required to complete the one-step cycle $\tau=\pi/(2g)$

Power delivered $P=2g\hbar\omega_0/\pi$

CONCLUSIONS

- **A resource theory for quantum and nano-scale heat engine:** a novel theoretical understanding of conversion of heat into work in quantum heat engines operating in the one-shot finite-size regime, and the role of inter-system correlations in such processes.
- **Engine with one-step cycle.**
- **Quantum superiority:** no fundamental trade-off between power and efficiency.
- **Carnot efficiency at maximum power.**
- **QHE based on quantum optics.**

REFERENCES

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