Preserving quantum correlations and coherence with non-Markovianity https://arxiv.org/abs/2106.13573

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Abstract

Open quantum systems exhibit a rich phenomenology, in comparison to closed quantum systems that evolve unitarily according to the Schrödinger equation. The dynamics of an open quantum system are typically classified into Markovian and non-Markovian [2], depending on whether the dynamics can be decomposed into valid quantum operations at any time scale. Since Markovian evolutions are easier to simulate, compared to non-Markovian dynamics, it is reasonable to assume that non-Markovianity can be employed for useful quantum-technological applications. Here, we demonstrate the usefulness of non-Markovianity for preserving correlations and coherence in quantum systems. For this, we consider a broad class of qubit evolutions, having a decoherence matrix separated from zero for large times, so that the evolution does not become simply unitary at long time scales. We focus on qubit systems, which is enough to demonstrate the main features we are interested in. While any such Markovian evolution leads to an exponential loss of correlations, non-Markovianity can help to preserve correlations even in the limit $t \to \infty$. For covariant qubit evolutions, we also show that non-Markovianity can be used to preserve quantum coherence at all times, which is an important resource for quantum metrology. We explicitly demonstrate this effect experimentally with linear optics, by implementing the required evolution that is non-Markovian at all times.

The Problem

Since covariant evolutions exhibit symmetry with respect to a given Hamiltonian, its eigenbasis provides a natural reference for defining quantum coherence. In case of two-level systems this corresponds to considering phase-covariant evolutions [1], which cover all dynamics that respect rotational symmetry about an axis in the Bloch representation, e.g. the z-axis, with the following decoherence matrix in the Pauli basis:

$$\gamma(t) = \begin{pmatrix} a & -ix & 0 \\ ix & a & 0 \\ 0 & 0 & f(t) \end{pmatrix}. \tag{1}$$

Note that the finction $f(t)$ can have any sign. The evolution of an initial qubit state $\rho_t = \Lambda_t \rho(\vec{r}_0) = \rho(\vec{r}_t)$ is given by

$$r_{1,2}(t) = \alpha(t) r_{1,2}(0), \tag{2a}$$
$$r_3(t) = \beta(t) r_3(0) - c(t). \tag{2b}$$

with $\alpha(t) = e^{-at-\int_0^t f(t)dt}$, $\beta(t) = e^{-2at}$, $c(t) = \frac{x}{a}(1-e^{-2at})$.
Then, we can write the Choi-Jamiołkowski (CJ) state of this evolution as

$$\Omega_t = \frac{1}{4}\begin{pmatrix} 1+\beta(t)-c(t) & 0 & 0 & 2\alpha(t) \\ 0 & 1-\beta(t)+c(t) & 0 & 0 \\ 0 & 0 & 1-\beta(t)-c(t) & 0 \\ 2\alpha(t) & 0 & 0 & 1+\beta(t)+c(t) \end{pmatrix}$$

For the resulting evolution Λ_t to be completely positive, and the final state to be entangled, we require: $(1-\beta(t))^2 < 4\alpha(t)^2 + c(t)^2 \leq (1+\beta(t))^2$.
It is straightforward to verify that the optimal choice of the function $f(t)$, saturating the previous condition, is

$$f(t) = -\frac{1}{2}a\left(1-\frac{x^2}{a^2}\right)\frac{\sinh 2at}{\cosh^2 at - \frac{x^2}{a^2}\sinh^2 at} \tag{3}$$

This function is always negative, that is why we call this evolution eternally non-Markovian.

Different resources

For the optimal choice of $f(t)$ as in Eq. (3), the negativity of the CJ state is given by

$$E(\Omega_t) = \frac{1}{2}e^{-2at}. \tag{4}$$

We see that the evolution Λ_t preserves entanglement for all finite times, as the CJ state is entangled in this case. However, Λ_t is entanglement breaking in the limit $t \to \infty$, as the CJ state becomes separable in this limit.
Interestingly, the mutual information does not vanish in the limit $t \to \infty$:

$$\lim_{t\to\infty} I(\Omega_t) = -\frac{1}{2}(p \log_2 p + (1-p)\log_2(1-p)) \tag{5}$$

with $p = \frac{1+\frac{x}{a}}{2}$. Note that part of this information is purely quantum discord.
In this case, we find the evolution which preserves quantum coherence for all finite times, including the limit $t \to \infty$. Interestingly, this dynamics converges to a map which has a 2-dimensional image, having finite coherence with respect to the reference basis.

$$C_{\ell_1}(t) = \frac{1}{2}C_{\ell_1}(0)\sqrt{(1+e^{-2at})^2 - \frac{x^2}{a^2}(1-e^{-2at})^2}. \tag{6}$$

As quantum coherence is a resource useful for quantum metrology, this dynamics allows us to estimate a parameter ω encoded in the unitary $U = e^{-i\omega\sigma_z}$, leading to non-zero quantum Fisher information even in the limit $t \to \infty$.

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Some plot

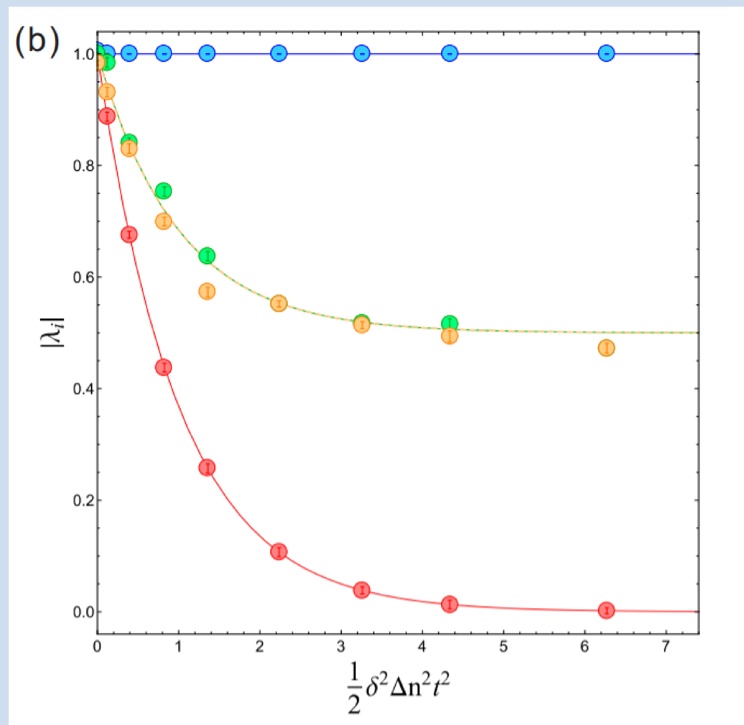

Figure 1. The dots represent the experimental values of the $\alpha(t)$ (green and yellow), $\beta(t)$ (red) and $1-c(t)$ (blue), for $x = 0$.

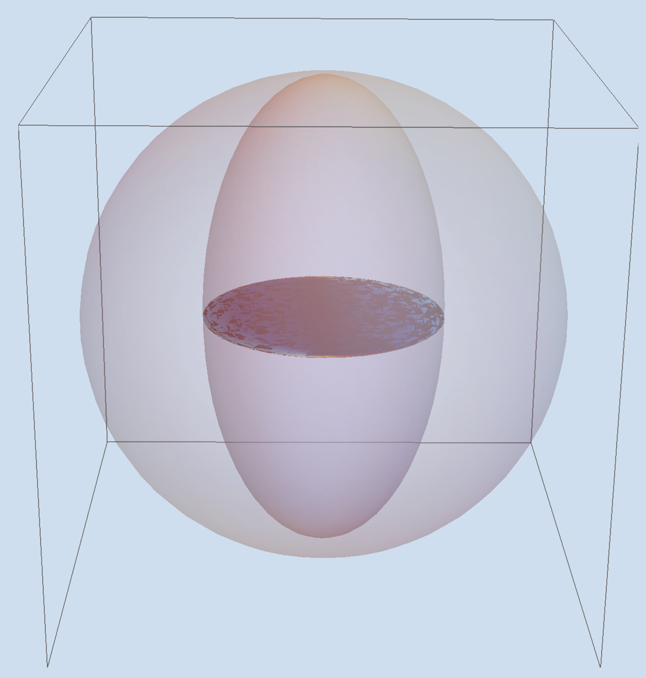

Figure 2. Bloch sphere evolution for $x = 0$, the final image is a disk inside the ellipsoid described by the complete positivity condition $4\alpha(t)^2 + c(t)^2 \leq (1+\beta(t))^2$.

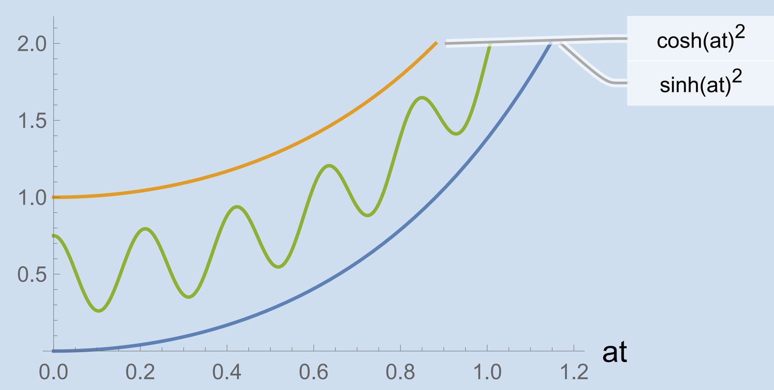

Figure 3. In green a generic $f(t)$ fulfilling the condition $(1-\beta(t))^2 < 4\alpha(t)^2 + c(t)^2 \leq (1+\beta(t))^2$, which converge to one function as $t \to \infty$.

References

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