Towards protecting quantum correlations in presence of noisy channel: the two-qubit case

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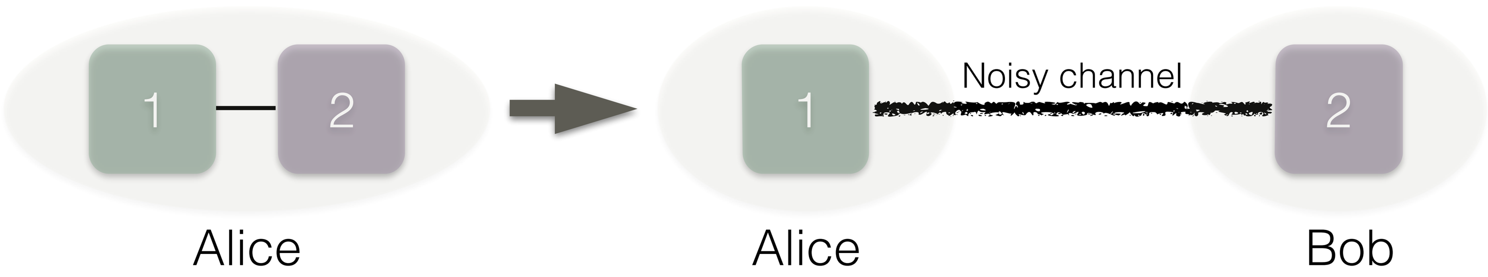
The inseparable nature of quantum correlations play an important role as resources in different information processing tasks. These resources are in general vulnerable in presence of environmental noise. We show that it is possible to obtain advantage for entanglement and steering by employing some selective measurements before or after the environmental interaction with the system. This prescription of finding the suitable measurement operators has been introduced based on the unitary evolution of the corresponding channel and hence not unique, which leaves us with different choices.

DECOHERENCE MODEL AND QUANTUM CORRELATION

We consider Generalised amplitude damping channel (GADC), Kraus operators of which are given as,

$$K_1 = \sqrt{\nu}\begin{bmatrix}1 & 0\\ 0 & \sqrt{\eta}\end{bmatrix};\ K_2 = \sqrt{\nu}\begin{bmatrix}0 & \sqrt{1-\eta}\\ 0 & 0\end{bmatrix}$$
$$K_3 = \sqrt{1-\nu}\begin{bmatrix}\sqrt{\eta} & 0\\ 0 & 1\end{bmatrix};\ K_4 = \sqrt{1-\nu}\begin{bmatrix}0 & 0\\ \sqrt{1-\eta} & 0\end{bmatrix}. \quad (1)$$



Alice performs a black box measurement at her own side producing an assemblage at Bob's end given by,

$$\sigma_B^{a|x} = Tr_A[(M_a^{\,x} \otimes \mathbf{I})\rho_{AB}] \quad (2)$$

Now, Bob performs a qubit measurement on the assemblage and hence producing a correlation $p(ab|xy)$. This correlation is said to be steerable if it cannot be decomposed as a LHS model. For quantifying the steerability we consider the violation of a steering inequality,
Analog CHSH inequality for steering:

$$\sqrt{\langle (A_0+A_1)B_0\rangle^2 + \langle (A_0+A_1)B_1\rangle^2} + \sqrt{\langle (A_0-A_1)B_0\rangle^2 + \langle (A_0-A_1)B_1\rangle^2} \leq 2 \quad (3)$$

where,

$$\langle A_x B_y\rangle = \sum_{a,b} (-1)^{a\oplus b} p(ab \mid xy) \quad (4)$$

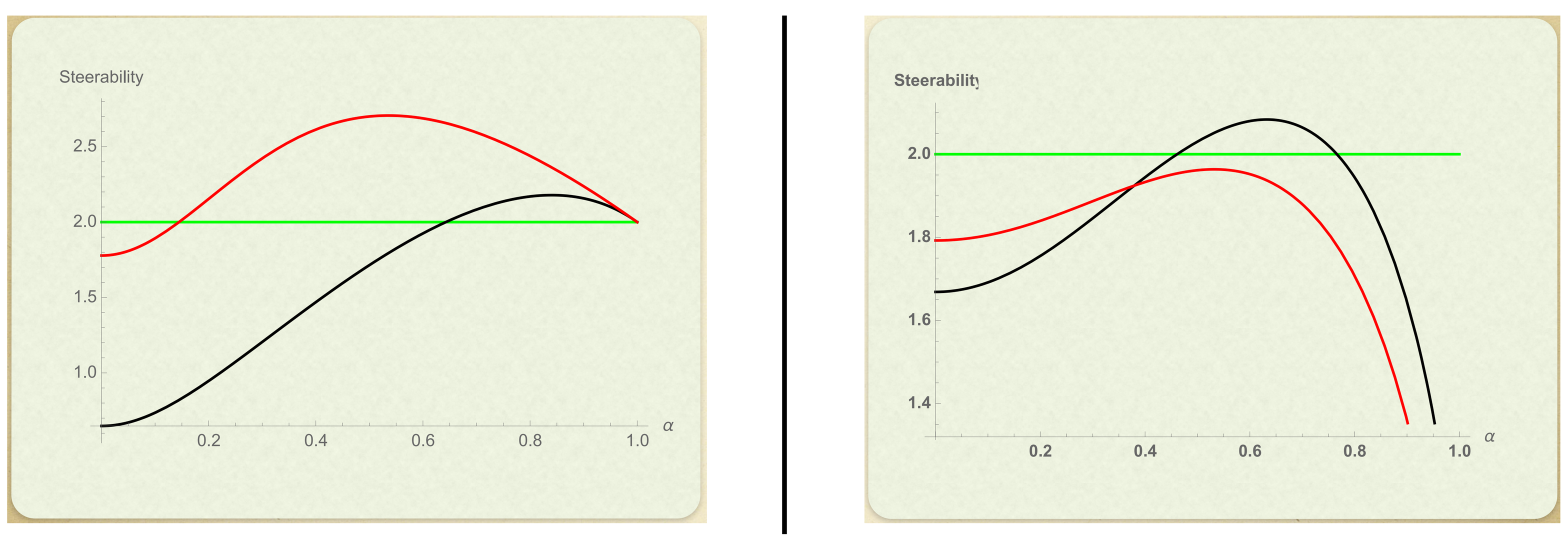
PARTIAL COLLAPSE MEASUREMENT

The detector detects the system with probability w if and only if the state of the second particle is in $|1\rangle$ $(= \begin{bmatrix}0\\1\end{bmatrix})$. Here we consider the measurement operator corresponding to the scenario when the system is not detected by the measuring apparatus. The measurement operator W_0 corresponding to this situation can be evaluated by using the relation, $W_1^\dagger W_1 + W_0^\dagger W_0 = \mathbf{I}$. Hence,

$$W_0 = |0\rangle\langle 0| + \sqrt{1-w}\,|1\rangle\langle 1| = \begin{bmatrix}1 & 0\\ 0 & \sqrt{1-w}\end{bmatrix}. \quad (5)$$

and the reverse measurement operator is,

$$R_0 = \begin{bmatrix}\sqrt{1-r} & 0\\ 0 & 1\end{bmatrix}. \quad (6)$$



(i) $\nu = 1$ i.e. ADC, (ii) $\nu = 0.234, \eta = 0.646$ i.e. GADC.

UNITARY EVOLUTION OF THE CHANNEL

Any general CPTP map has an operator-sum representation (or, Kraus representation) expressed as,

$$\mathcal{N}(\mathcal{A}) = \sum_{i=1}^{M} L_i \mathcal{A} L_i^{\dagger} \quad (7)$$

with the Kraus operators L_i satisfying the condition, $\sum_{i=1}^{M} L_i^{\dagger} L_i = \mathbf{I}$. Alternatively, for every $\mathcal{A}$ (thus, $\mathcal{A}$ can also be a density matrix of S), one can write,

$$\mathcal{N}(\mathcal{A}) = Tr_B[U_{SB}(\mathcal{A}\otimes |1\rangle_B\langle 1|)U_{SB}^{\dagger}]. \quad (8)$$

Our aim is to find a $dMxdM$ unitary matrix U_{SB} which corresponds to the map such that, $L_i = {}_B\langle i|\,U_{SB}\,|1\rangle_B, \forall i = 1, 2, ..., M$, with d being the dimension of the system Hilbert space. Hence, the (αi , $\beta 1$) -entry of the matrix U_{SB} can be obtained in the following way,

$$\begin{aligned} u_{\alpha i,\beta 1} &\equiv ({}_S\langle\alpha|\otimes {}_B\langle i|)U_{SB}(|\beta\rangle_S\otimes|1\rangle_B) \\ &= {}_S\langle\alpha|\,L_i\,|\beta\rangle_S,\ \forall \alpha,\beta = 1, 2, ..., d. \end{aligned} \quad (9)$$

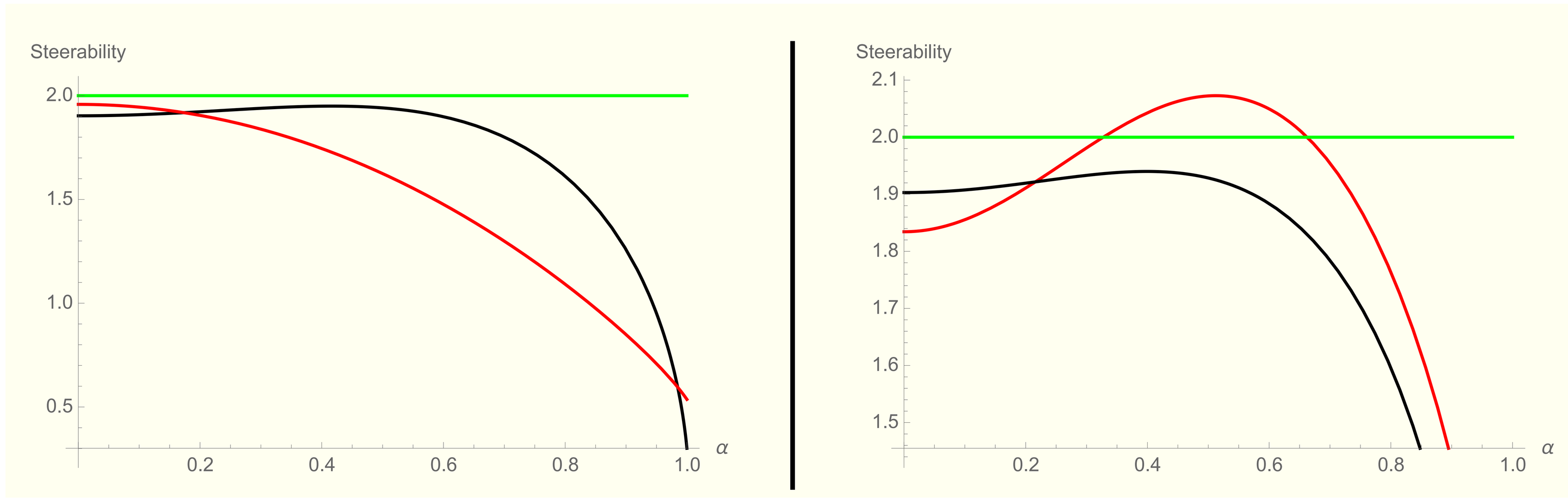
- All the column vectors of U_{SB} should be orthogonal to each other.
- The individual columns must be normalised.
- The unitary should reproduce identity map in case of GADC, for $\nu = \eta = 1$.

The Kraus operators corresponding to the channel described by the inverse unitary,

$$Tr_B[U_{SB}^{\dagger}(\sigma_S\otimes|1\rangle_B\langle 1|)U_{SB}] = \sum_{i=1}^{4} J_i \sigma_S J_i^{\dagger}, \quad (10)$$

with, $J_i = {}_B\langle i|\,U_{SB}^{-1}\,|1\rangle_B$ for $i = 1, 2, 3, 4$. One set of measurement operators corresponding to the inverse unitary,

$$J_1^{(1)} = \begin{bmatrix}\sqrt{\nu} & 0\\ 0 & \sqrt{\eta\nu}\end{bmatrix};\ J_2^{(1)} = \begin{bmatrix}-\frac{\sqrt{\eta-\eta\nu}}{\sqrt{-\nu\eta+\eta+\nu}} & 0\\ 0 & -\sqrt{\eta-\eta\nu}\end{bmatrix}$$
$$J_3^{(1)} = \begin{bmatrix}0 & -\sqrt{1-\eta}\\ 0 & 0\end{bmatrix};\ J_4^{(1)} = \begin{bmatrix}0 & 0\\ -\frac{\sqrt{(\eta-1)(\nu-1)\nu}}{\sqrt{-\nu\eta+\eta+\nu}} & 0\end{bmatrix}. \quad (11)$$



(i) Employing partial collapse measurement, (ii) Employing the technique of unitary dilation. ($\nu = 0.054$, $\eta = 0.551$)

CONCLUSION AND FUTURE ASPECTS

- We introduce a general method to preserve certain types of quantum correlations in presence of environmental noise.
- The prescription is implementable whenever the dynamics of the channel is known.
- The unitary dynamics is not unique.
- Our aim is to choose such an U_{SB} (as well as $\sigma_B^{(0)}$) such that:
i) U_{SB} becomes $\mathbf{I}_S\otimes\mathbf{I}_B$ whenever $\mathcal{N}$ becomes the identity channel.
ii) $U_{SB}(\rho_S\otimes\sigma_B^{(0)})U_{SB}^{\dagger}$ is close to a product state of the form $Tr_B[U_{SB}(\rho_S\otimes\sigma_B^{(0)})U_{SB}^{\dagger}]\otimes\sigma_B^{(1)}$ where $\sigma_B^{(1)}$ is fixed state of the ancilla.

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