Predicting **work fluctuations** from heat measurements in **coherent systems**

Quantum mean-square predictors of work

Marcelo Janovitch, Gabriel T. Landi

1 Intro

For incoherent quantum systems the *two-point measurement* (TPM) [1] protocol entails the statistics of heat and work. Yet, once applied to initially coherent systems, it erases any coherence in the energy eigenbasis. We propose a partial solution: Giving up measuring the internal energy of the system (S), we develop a predictor of work based on measuring the heat exchanged with the environment (E), while maintaining system's coherence untouched.

2 Formal framework

The energy balance of $S+E$ Hamiltonian

$$H(t) = H_S(t) + H_E + H_{SE}, \tag{1}$$

implies the 1st law

$$\langle W \rangle = \langle Q \rangle + \langle \Delta U \rangle . \tag{2}$$

In particular for steady-states, $\langle \Delta U \rangle = 0$ and *average* heat exactly describes *average* work.

- Even for steady-states, ΔU generally fluctuates;
- In general, we'd need ΔU and Q to infer W, but measuring ΔU destroys system's coherence;
- We can measure through the TPM $Q(\gamma) = \boldsymbol{\mu}' - \boldsymbol{\mu}$, with $\gamma \equiv (\boldsymbol{\mu}, \boldsymbol{\mu}')$ bath's two-time energies;
- Statistical prediction: Finding the function $\mathcal{W}_{\text{opt}}[Q]$ which minimizes the mean-square error w.r.t. $P(W, Q, \Delta U)$;
- $P(W, Q, \Delta U)$ is given by a quantum Bayesian network distribution [2];

Main result [3]: The predictor of work which minimizes the Bayesian network mean-squared error is given by

$$\mathcal{W}_{\text{opt}}(\gamma) = Q(\gamma) \; + \; \frac{\left\langle M_\gamma^\dagger H_S' M_\gamma - M_\gamma^\dagger M_\gamma \mathbb{D}(H_S) \right\rangle_{\rho_S}}{P(\gamma)}, \tag{3}$$

- ρ_S is the initial state of S;
- $\mathbb{D}(\bullet) = \sum_\alpha |\psi_\alpha\rangle\langle\psi_\alpha| \bullet |\psi_\alpha\rangle\langle\psi_\alpha|$ is the full dephasing operator in the basis of ρ_S (not the energy basis!);
- M_γ are Kraus components of the open-system dynamics of S (conditioned dynamics);
- $P(\gamma)$ is the TPM probability s.t. $P(Q) = \sum \delta[Q - (\boldsymbol{\mu}' - \boldsymbol{\mu})] P(\gamma)$;

3 Example: SSDB Heat Engine

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \kappa f_h D[L_{31}] + \kappa(1-f_h) D[L_{13}] + \kappa f_c D[L_{21}] + \kappa(1-f_c) D[L_{12}]. \tag{4}$$

with $L_{ij} = |i\rangle\langle j|$, D Lindblad operators and

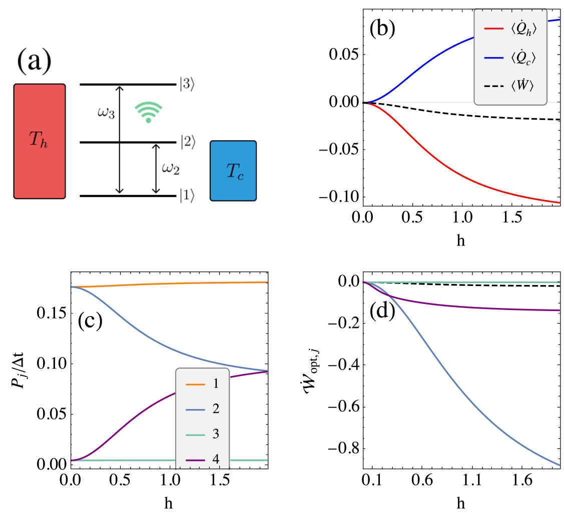
$$H_S(t) = \omega_3 |3\rangle\langle 3| + \omega_2 |2\rangle\langle 2| + V(t), \tag{5}$$

$$V(t) = h(|2\rangle\langle 3| \, e^{-i\omega t} + e^{+i\omega t} \, |3\rangle\langle 2|). \tag{6}$$

We build the predictor with the jump unravelings

$$M_1 = \sqrt{\kappa f_h \Delta t} \; L_{31}, \qquad M_2 = \sqrt{\kappa(1-f_h)\Delta t} \; L_{13}, \tag{7}$$

$$M_3 = \sqrt{\kappa f_c \Delta t} \; L_{21}, \qquad M_4 = \sqrt{\kappa(1-f_c)\Delta t} \; L_{12}, \tag{8}$$



4 Conclusion

We develop a tool to predict work fluctuations conditioned on heat measurements. Our result can be seen as a first law at the level of *heat* trajectories. We use it to predict work realizations in the SSDB heat engine, whose steady-state is coherent.

References

[1] P. Talkner, E. Lutz, and P. Hänggi, "Fluctuation theorems: Work is not an observable," *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, vol. 75, no. 5, p. 050102, May 2007. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevE.75.050102.

[2] K. Micadei, G. T. Landi, and E. Lutz, "Quantum Fluctuation Theorems beyond Two-Point Measurements," *Physical Review Letters*, vol. 124, no. 9, p. 90602, 2020. [Online]. Available: https://doi.org/10.1103/PhysRevLett.124.090602.

[3] M. Janovitch and G. T. Landi, "Quantum mean-square predictors of work," Apr. 2021. [Online]. Available: http://arxiv.org/abs/2104.07132.

Universität Basel IFUSP Instituto de Física da USP CAPES Quantum Thermodynamics Quantum Transport